Random Vibration Simulation with Abaqus

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Executive Summary

The concepts associated with performing random vibration simulations are presented in terms of the random response procedure available within the Abaqus finite element simulation software. The paper begins with some general background information on the topic of random vibration. The equations that form the basis for a random response simulation are then presented in matrix notation. The concepts of "correlated" and "uncorrelated" random excitations are discussed and illustrated. The random response procedure is demonstrated by solving a series of simple beam problems.

Random response procedures are not supported by the 2017x release of the "Linear Dynamics Scenario Creation" app of the Dassault Systémes 3DEXPERIENCE Platform. However, the Abaqus Power'By apps allow random response simulations to be performed on the 3DEXPERIENCE Platform. These apps are located in the V+R quadrant of the platform compass with the designations "Run Abaqus/CAE" and "Abaqus Study".

This paper is intended for engineers and analysts responsible for simulating noise and vibration in mechanical systems.

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1. Introduction

The Abaqus finite element simulation software can be used to analyze the linear dynamic response of structural systems undergoing excitations that are random in time. The random designation signifies that the value of an excitation cannot be predicted in a deterministic manner. However, if the excitation exhibits some degree of statistical regularity it is possible to describe the excitation in a statistical sense. The same statistical measures used to describe the random excitation can also be used to describe the structure's dynamic response. Examples of random excitations include turbulence loading on an aircraft; car loadings induced by road surface imperfections and earthquake ground motions. The statistical measures of an excitation and the structural response variables that are of most importance include:

- Expected (Mean) Value
- Mean Square Value
- Root Mean Square (RMS)
- Power Spectral Density (PSD)
- Autocorrelation and Cross-Correlation

Simulation of the dynamic response of structures under random loading is referred to in the Abaqus documentation as a Random Response analysis. The purpose of this paper is to provide the reader with an overview of random response concepts and how they relate to the Abaqus random response simulation procedure. Several good sources on the subject of random vibration are listed in the reference section of this paper. Also, Internet searches on the terms "random response" and "random vibration" will produce a wide range of sources including books, scholarly articles, PhD theses, and articles from engineering publications.

1.1. Random Response Basic Concepts

The Abaqus random response procedure incorporates some basic assumptions. First, the time associated with the excitation and response transients are "long" so that any short term transients can be ignored. Second, the excitation and response are stationary in time, meaning that relevant statistical properties do not vary with time. For example, if $z(t)$ is the sample being considered, then any statistical measure of $z(t)$ must have the same value regardless of what time origin is used to compute the statistic. Third, the excitation and response are ergodic, meaning the time averages of several excitation or response samples are identical. Figure 1 is an example of a typical random sample. Following are some definitions of the basic statistical measures that are used to describe a random sample.

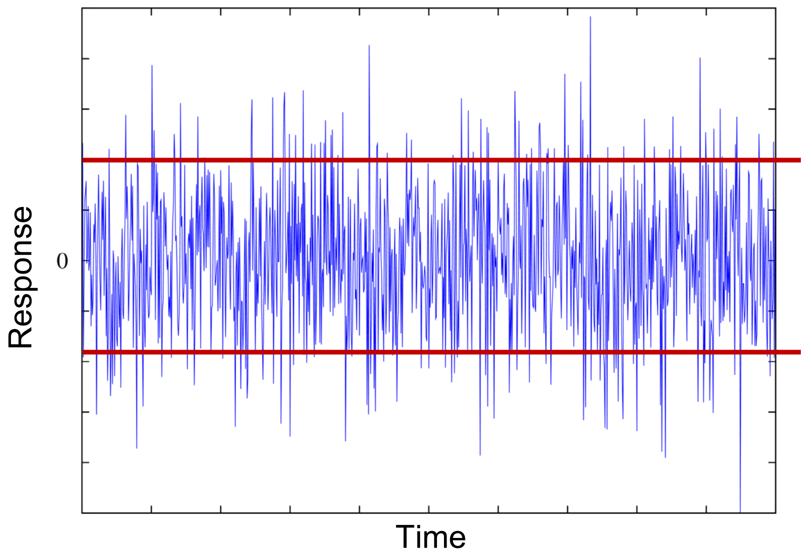


Figure 1: Random Sample

The Expected Value, also referred to as the Mean Value of a random sample *z*(*t*) is the time averaged value of the sample.

$$E(z) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} z(t)dt \qquad \text{(Eq. 1)}$$

The Variance measures the time averaged squared difference between the random sample and its Expected Value.

$$\sigma_z^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |z(t) - E(z)|^2 \, dt \qquad \text{(Eq. 2)}$$

The Abaqus random response procedure assumes the structural vibrations due to random excitations are sufficiently small to be classified as linear. It also assumes the Expected Value of the random excitation can be related to a static Base State (i.e., preload). The random response of a structure is solved as a linear perturbation about a Base State, such that the random excitation is expressed with an Expected Value of zero, with the output response variables being determined relative to the Base State.

$$E(z) = 0 \qquad \text{(Eq. 3)}$$

Therefore, with respect to the Base State, the Variance equals the Mean Square Value.

$$\sigma_z^2 = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |z(t)|^2 \, dt \qquad \text{(Eq. 4)}$$

The Root Mean Square (RMS) value of a random sample is the square root of the Variance.

$$\sigma_z = \sqrt{\sigma_z^2} \qquad \text{(Eq. 5)}$$

The RMS value of the random sample represents one Standard Deviation (SD) of the sample's probability density function. The probability density function provides a statistical measure of the probability that the value of the sample will be within specified limits at any given time, for example the red lines shown in Figure 1. When a random sample's probability density function has the form of a Gaussian (normal) distribution, the sample's value is within the Standard Deviation limits (+/- $1\sigma_z$) 68.3% of the time. The sample's value will be within the two Standard Deviation limits (+/- $2\sigma_z$) 95.4% and three Standard Deviation limits (+/- $3\sigma_z$) 99.7% of the time. Reference 1 indicates that the Gaussian distribution occurs frequently in nature and is characterized by a bell shaped curve.

Random samples contain a continuous distribution of contributing frequencies. The Power Spectral Density (PSD) curve, S_z, expresses the Mean Square Value per unit frequency as a function of frequency. The term "Power" refers to the concept of a Mean Square Value, the term "Spectral" refers to a frequency domain function, and the term "Density" indicates that the values are per a unit interval of frequency. For example, the units associated with the PSD curve of a force excitation would be F^2/Hz. The Mean Square Value (Variance) of a random sample $z(t)$ can be expressed in terms of the sample's PSD as:

$$\sigma_z^2 = \int_0^\infty S_z(f)\,df \qquad \text{(Eq. 6)}$$

In a practical sense the frequency range of the excitation and resulting structural response are limited and the integration is carried out to the maximum frequency of interest. Also, performing the integration over a discrete frequency range defined by lower and upper limits provides the range's contribution to the total Mean Square Value.

[1]Thomson, W.T., *"Theory of Vibration with Applications,"* Prentice-Hall, Inc., 1972.

The Autocorrelation of a random sample $z(t)$ is the Expected Value of the product of the sample and itself when shifting the time origin.

$$R_z(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} z(t)z(t+\tau)dt \quad \text{(Eq. 7)}$$

The Cross-Correlation of a random sample $z(t)$ with a random sample $y(t)$ is the Expected Value of the product of the samples with a shift in the time origin of the second sample.

$$R_{zy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} z(t)y(t+\tau)dt \quad \text{(Eq. 8)}$$

The Fourier transform of the Autocorrelation function of a random sample is equal to the sample's Power Spectral Density (PSD) curve. Likewise, the Fourier transform of the Cross-Correlation function of two random samples is equal to the Cross-Correlation PSD. The concept of PSD curves being derived from Autocorrelation and Cross-Correlation functions of random samples is utilized when evaluating the random response of structures undergoing multiple simultaneous excitations. This is discussed further in the section that follows when considering the governing statistical equations presented in matrix format.

The ability to describe a random excitation statistically in terms of a Power Spectral Density allows the random response simulation to be performed in the frequency domain rather than the time domain. One example of a frequency domain simulation is the Abaqus Steady State Dynamic (SSD) procedure which is used to compute discrete output quantities due to harmonic excitations as a function of frequency. The ratio of an output quantity to an applied excitation is referred to as a transfer function. In most applications the harmonic excitation is meant to represent a unit loading (i.e., force, acceleration, etc.) such that the solution output represents the transfer function. Transfer functions are generally complex, and as such are capable of characterizing the phase relationship between input and output. Knowing the transfer function between harmonic input and harmonic output allows the Power Spectral Density of an output quantity to be determined given the excitation PSD. For simple systems with single force excitations random response output can often be derived via post-processing of a Steady State Dynamic simulation. Consider the following solution sequence for determining the response of an output variable given a single random excitation.

- A PSD curve S_z for a random force excitation is provided as a real-valued design specification with units of F^2/Hz.

- A frequency response transfer function $H_x(f)$ representing the output response, x, due to a unit force excitation, z, is obtained. The transfer function can be determined either experimentally or from an Abaqus harmonic Steady State Dynamic simulation.

- The PSD curve for the random response output variable is obtained by multiplying the Excitation PSD by the square of the transfer function magnitude. This operation is performed at each frequency solution point within the range defined for the Excitation PSD.

$$S_x(f) = H_x^2(f) S_z(f) \quad \text{(Eq. 9)}$$

- The Mean Square Value (Variance) of the random response output variable can then be estimated by numerically integrating the response PSD (Eq. 6).

- The RMS response (one standard deviation) for the random output variable is then obtained as the square root of the Mean Square Value (Eq. 5).

The random response approach described above works well for systems undergoing a single excitation for which only limited response output is required. However, for more complicated systems with multiple excitations and/or requirements for large amounts of output the approach described above is inadequate. To address this issue, the Abaqus random response procedure utilizes a modal superposition technique in which the excitations are projected onto the modal space. The modal projection is also applied to the excitation PSDs and modal transfer functions are used to obtain the generalized modal response PSDs. The required element and/or nodal responses are then derived from the modal results.

The Abaqus random response solution must be preceded by a frequency extraction about the preloaded Base State. The extracted modes are then available for use in the modal superposition technique utilized by the random response procedure. Even though the Abaqus random response procedure utilizes modal superposition, considering the governing matrix equation expressed in terms of the applied excitations and required output provides the best format for understanding the general concepts as well as how the analyst interfaces with Abaqus.

1.2. Random Vibration Governing Matrix Equation

Consider the random vibration problem as defined by the following matrix equation:

$$\underset{n \times n}{[S_o(f)]} = \underset{n \times m}{[H(f)]}\ \underset{m \times m}{[S_e(f)]}\ \underset{m \times n}{[H^*(f)]^T} \quad \text{(Eq. 10)}$$

(f) indicates these matrices are a function of frequency.

m is the number of excitations.

n is the number of response output variables to be generated.

* designates the complex conjugate.

$[\,]^T$ transposed matrix

$[S_e(f)]$ is the Power Spectral Density (PSD) Excitation matrix for ***m*** random inputs to the structural system. It is referred to in the Abaqus documentation as the Cross-Spectral Density matrix.

$[S_o(f)]$ is the Response Power Spectral Density matrix for the ***n*** output variables.

$[H(f)]$ is a matrix of transfer functions for *n* response output variables resulting from *m* excitations. Each row contains the transfer functions for a single response output variable resulting from the *m* excitations.

The term "excitation" as used in Eq.10 represents a load condition of unit magnitude which is used to generate the corresponding transfer functions. Therefore, each transfer function represents the harmonic frequency response due to a unit excitation. The magnitudes of the random excitations are characterized by the PSD Cross-Spectral Density matrix. An excitation as represented in Eq. 10 can be a nodal force component, an applied pressure, a base motion, or a set of forces that act in unison. The manner in which unit excitations are defined within Abaqus will be discussed in the sections that follow.

The analyst provides the Abaqus keyword data (type: model and history) required to specify the Cross-Spectral Density matrix (excitation PSD). The "unit excitations" are defined within the random response simulation step (history data). The Abaqus finite element model and the excitations are sufficient to create the needed transfer functions.

The Response PSDs for the required output are then generated by the Abaqus solver by means of the modal superposition technique described earlier.

In the most general mathematical sense, the Cross-Spectral Density, Transfer Function, and Response PSD matrices can all be complex functions of frequency. The Abaqus random response procedure supports fully complex solutions; however, it is often common practice to define the Cross-Spectral Density matrix as being real-valued, resulting in Response PSDs that will also be real-valued.

The diagonal terms of the excitation Cross-Spectral Density matrix are the Autocorrelation PSDs, which are equal to the Fourier transform of Autocorrelation excitation time histories. The off-diagonal terms are the Cross-Correlation PSDs, which are the Fourier transforms of Cross-Correlation excitation time histories. The Autocorrelation PSDs are always non-zero. The Cross-Correlation PSDs for excitations that are statistically independent will be zero. The excitations in this case are referred to as being uncorrelated; that is, they have no relationship to each other. Excitations that act in unison are statistically dependent. They are referred to as being correlated and will have non-zero Cross-Correlation PSDs. Multiple excitations that are correlated (i.e., act in unison), can always be combined to act as a single excitation. Excitations that have some degree of commonality and/or do not have the same Auto-correlation generated PSDs may also have non-zero off-diagonal terms. In this case the excitations are neither fully correlated nor uncorrelated, and the Abaqus UCORR user subroutine can be used to complete the definition of the off-diagonal terms of the Cross-Spectral Density matrix

The terms of the Response PSD matrix correspond to variables such as force, displacement, velocity, acceleration, stress components and strain components. Each diagonal term of the Response PSD matrix is equal to the Fourier transform of the variable's time response Autocorrelation. Off-diagonal terms are equal to the Fourier transform of the time response Cross-Correlation between pairs of output variables. The Abaqus Theory Guide states, "We might also compute the cross-spectral densities between variables. These are usually not of interest, and Abaqus/Standard does not provide them. They might be needed if the analysis involves obtaining results that, in turn, will define the loading for some other system. For example, the response of a building to seismic loading might be used to obtain the motions of the attachment points for a piping system in the building so that the piping system can then be analyzed. The only option would be to model the entire system together."

Excitations are defined via forces/moments, connector forces/moments, pressure loading or base motions within an Abaqus random response simulation. Base motions are converted to inertial forces and a pressure load is converted to discrete nodal forces. Abaqus uses the concept of a Load Case to identify random excitations. The Load Case number assigned to an excitation is included in the definition of each load contributor (i.e., nodal force/moment). When defining excitations via Load Cases, the governing matrix equation (Eq. 10) can be rewritten in terms of nodal loads and their associated Transfer Functions. By default, Abaqus will treat loads assigned to the same Load Case as being correlated, for which diagonal and off-diagonal terms will contribute to the Cross-Spectral Density matrix as expressed with respect to nodal loads. Abaqus also provides the option to treat the loads within a Load Case as being uncorrelated, for which only diagonal terms will contribute to the nodal load based Cross-Spectral Density matrix. Separate Load Cases are always processed as being independent excitations. That is, each Load Case will contribute to the Cross-Spectral Density matrix but no cross-correlation exists between Load Cases. Therefore, when multiple excitations are determined to be dependent (correlated), they should be combined into a single Load Case.

An excitation Cross-Spectral Density matrix expressed with respect to nodal loads is generated for each Load Case as the product of a complex valued frequency dependent scalar function and a spatial coupling matrix that identifies the active nodal loads and their correlation condition. The spatial coupling matrix, referred to in Abaqus as a Cross-Correlation matrix, is generated based upon the load data and correlation condition. The system's nodal load based Cross-Spectral Density matrix is assembled by a matrix summation over all the excitation Load Cases. In the example shown below, there are three Load Cases applied to nodes that have a single degree-of-freedom. Each Load Case has a characteristic PSD scalar function, PSD_1, PSD_2, PSD_3, and associated spatial coupling (Cross-Correlation) matrices. Excitation 1 has a single force at node 1, excitation 2 has uncorrelated forces at both nodes 2 and 3, and excitation 3 has correlated forces at both nodes 1 and 3. The spatial coupling matrices for the three excitations contain unit values on the diagonals corresponding to the nodal loads and unit off-diagonal values for load pairs in correlated Load Cases. No cross-correlation occurs between loads of separate Load Cases.

$$[S_e(f)] = PSD_1(f)\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}_1 + PSD_2(f)\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}_2 + PSD_3(f)\begin{bmatrix}1 & 0 & 1\\ 0 & 0 & 0\\ 1 & 0 & 1\end{bmatrix}_3$$

Load Case 1: Force at node 1 with PSD_1 scalar function

Load Case 2: Forces at node 2 and node 3 (uncorrelated) with PSD_2 scalar function

Load Case 3: Forces at node 1 and node 3 (correlated) with PSD_3 scalar function

Load Cases consisting of a single nodal load only produce a diagonal contribution to the nodal load based Cross-Spectral Density matrix. Load Cases consisting of multiple uncorrelated loads only produce corresponding diagonal contributions. A Load Case consisting of multiple correlated loads produces both diagonal and off-diagonal contributions to the nodal load based Cross-Spectral Density matrix. Note, by default; the spatial coupling Cross-Correlation matrices contain unit values, and not the magnitudes of the nodal loads that constitute the Load Case. The magnitudes of the Load Case nodal loads are utilized when generating the required transfer functions. Therefore, the nodal loads that constitute a Load Case represent a unit excitation for which the corresponding PSD complex scalar function characterizes the random excitation. When defining a Cross-Spectral Density matrix that includes some partial or weak correlation, the UCORR subroutine can be used to assign weighting factors to a Load Case's Cross-Correlation matrix.

1.3. Matrix Equation Example

The 2-dimensional example that follows illustrates how the general PSD matrix equation (Eq. 10) is expanded and processed in terms of the Load Case equivalent nodal loads. The problem is of a circular section cantilever beam 62.5 units long and 1 unit in diameter that is located within the global X-Y plane. The random response solution uses a frequency range of 2 to 2002 Hz. Density (1.0e-14) and elastic properties (E=932660.0, v=0.45) are set to produce a fundamental mode in excess of 30 kHz so that the transfer functions, which are stiffness controlled in the designated frequency range, will be real-valued and constant. The excitation Load Case is a unit force applied at the free end and directed transverse to the beam's axis. The PSD Scalar function for the excitation represents a white noise condition. It is real-valued and set at a constant of 0.0005 F^2/Hz. Two orientations in the global X-Y plane as shown in Figure 2 are simulated. Position-A has the beam aligned with the global X axis and Position-B has the beam rotated downward by 45 degrees.

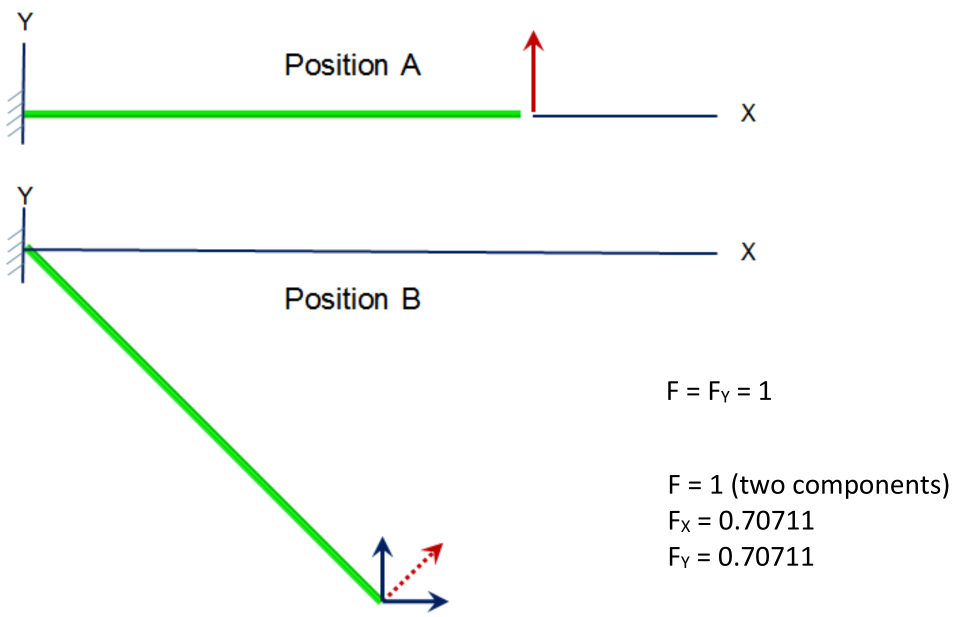


Figure 2: Simple Beam Test Simulations

The output of interest is the stress on the top of the beam in the element at the fixed end. For Position-A, the excitation Load Case consists of a unit force in the Y direction. The stress transfer function for this unit Y force is -636.62 over the frequency range of interest. The solution for the stress output PSD is trivial, given by:

$$[S_\sigma] = [H_\sigma](PSD_1\,[1])\ [H_\sigma{}^*]^T = (-636.62) * 0.0005 * (-636.62) = 202.64$$

Integrating the stress output PSD over the 2 - 2002 Hz frequency range and taking the square root produces an RMS stress of 636.6.

For Position-B the unit excitation Load Case consists of X and Y force components with a vector sum that is transverse to the beam with unit magnitude. The X and Y force components must retain their relative magnitudes and phasing at all times in order to represent a transverse force, therefore, they are correlated (dependent). With correlated X and Y forces, the excitation Cross-Spectral Density matrix is the product of the PSD scalar function and a Cross-Correlation matrix with unit values for all terms. Correlated X and Y force components of +0.70711 represent the unit excitation. The X and Y loading stress transfer functions in the 2 – 2002 Hz range can be obtained from static solutions since the fundamental mode is in excess of 30 kHz. The X and Y transfer functions are -317.6734 and -318.9466, respectively. The solution for the stress output PSD is given by:

$$[S_\sigma] = [H_{\sigma X}\ \ H_{\sigma Y}]\left[PSD_1\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right]\begin{bmatrix}H_{\sigma x}\\ H_{\sigma Y}\end{bmatrix}$$

$$[S_\sigma] = [-317.6734\ , -318.9466]\left[0.0005\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right]\begin{bmatrix}-317.6734\\ -318.9466\end{bmatrix} = 202.64$$

RMS stress = 636.6 (bending deformation response; 2 – 2002 Hz)

When changing from a transverse excitation to an axial excitation for Position B, the Y force component of the excitation is negative with a corresponding stress transfer function of +318.9466. The stress output PSD for an axial excitation is given by:

$$[S_\sigma] = [-317.6734,\quad 318.9466]\left[0.0005\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right]\begin{bmatrix}-317.6734\\ 318.9466\end{bmatrix} = 8.104\mathrm{e}^{-4}$$

RMS stress = 1.27 (axial deformation response 2 – 2002 Hz)

If the Position-B Excitation Load Case specifies independent X and Y force components, then the system has uncorrelated forces and the resulting Cross-Correlation matrix will have off-diagonal terms equal to zero. In this case, the sign of the excitation force components does not have an influence on the solution. Therefore, only the magnitudes of the transfer functions are required since there is no coupling via the Cross-Correlation matrix.

$$[S_\sigma] = [\,H_{\sigma X} \;\; H_{\sigma Y}] \left[PSD_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} H_{\sigma x} \\ H_{\sigma Y} \end{bmatrix} = PSD_1[\,H_{\sigma X}^2 + H_{\sigma Y}^2]$$

$$[S_\sigma] = [\,-317.6734 \quad 318.9466] \left[0.0005 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{bmatrix} -317.6734 \\ 318.9466 \end{bmatrix} = 101.32$$

RMS stress = 450.2 (uncorrelated force components; 2 – 2002 Hz)

2. Abaqus Random Response Procedure

The Abaqus random response procedure uses modal superposition techniques to characterize a structure's response. This requires that a frequency extraction solution step precede the random response step. The governing PSD matrix equation is transformed from the full set of nodal loads and requested output response variables to a set of modal excitations and modal responses. The transfer functions of the transformed system correspond to the generalized modal coordinates (i.e., mode scaling factors). The requested response variable PSDs are then recovered from the modal responses. From the viewpoint of setting up a random response simulation, the solution process can be thought of as using modal superposition to obtain the transfer functions needed to compute the response PSDs.

For a system with no damping, the transfer function solutions would become unbounded at the system's natural frequencies. Therefore, the structural system being simulated must include damping. Global damping and all of the mode based damping models available in Abaqus can be used, namely; modal, Rayleigh, composite or structural. However, the material damping projection approach associated with using the SIM architecture is not available with the random response procedure.

Random response output can be generated for either node or element based variables at the driving frequencies specified by the analyst for generating the modal response transfer functions. Output requests for field or history data associated with scalars (i.e., connector force), vectors (i.e., displacement, force, moment) and tensors (i.e., stress, strain) represent Power Spectral Density values. Preceding the output variable identifier with the letter R will request RMS (Root Mean Square) values. For example, the output request designated as "RS" provides RMS values of the stress components. The only tensor invariant for which PSD and RMS values are available is Von Mises stress.

Von Mises stress random output is calculated during post-processing by the Abaqus/CAE Visualization module via the method developed by Segalman, et al.[2] To obtain Von Mises random output during an Abaqus post-processing session, the preceding frequency extraction step must have included a stress (S) output request for the extracted modes. Examples of some random response output variable identifiers include:

- U (U1, U2, U3) = PSD displacement components
- RU (RU1, RU2, RU3) = RMS displacement components
- S (S11, S22, etc.) = PSD stress components
- RS (RS11, RS22, etc.) = RMS stress components
- RF (RF1, RF2, RF3) = PSD reaction force components
- RRF (RRF1, RRF2, RRF3) = RMS reaction force components

2.1. Abaqus Keywords for Random Response

As discussed in Section 1.2, the excitation Cross-Spectral Density matrix is defined by the analyst as the product of a frequency dependent complex-valued scalar function and a spatial coupling matrix referred to as the Cross-Correlation matrix. The scalar function represents the magnitude (and phase if complex) of an excitation's PSD while the Cross-Correlation matrix identifies the nodal loads and their correlation status. The PSD scalar functions are defined as model data via the *PSD-DEFINITION keyword. Cross-Correlation matrices are generated with the *CORRELATION keyword, which is placed within the random response step (history data). Each *CORRELATION keyword refers to a single PSD scalar function and one or more excitation Load Cases.

Excitation Load Cases are defined within a random response step via concentrated loads (*CLOAD), distributed loads (*DLOAD, *DSLOAD), connector loads (*CONNECTOR LOAD) and base motions (*BASE MOTION). Excitation Load Cases that are created, but not used by a *CORRELATION keyword will produce an error during the data check portion of the simulation.

[2]Segalman, D. J., C. W. G. Fulcher, G. M. Reese, and R. V. Field, Jr., "*An Efficient Method for Calculating RMS Von Mises Stress in a Random Vibration Environment*," Sandia Report, SAND98-0260, 1998.

Load Cases can utilize any identification number with the following exceptions:

- Pressure based loads (*DLOAD, *DSLOAD) must utilize Load Case 1.
- Pressure loads cannot be combined with any other loads within Load Case 1.
- Pressure loads (Load Case 1) should be designated as correlated.
- Load Case 1 can be used for other load types if pressure loads are not defined.

Performing a random response simulation in Abaqus requires a frequency extraction analysis step (*FREQUENCY) followed by the *RANDOM RESPONSE analysis step. Both types of analysis steps are linear perturbation steps. The frequency extraction step can be preceded by a general nonlinear solution in order to introduce a static preload condition. The *RANDOM RESPONSE keyword identifies the solution frequencies used to create the needed transfer functions, and therefore, the required output response PSDs. It utilizes no parameters and has the following data line format:

> *RANDOM RESPONSE
> *freq. lower limit, freq. upper limit, number of evaluation points, bias, frequency scale*

A sufficient number of solution frequencies are needed to define the output PSDs over the desired frequency range so that when integrated accurate RMS values can be obtained. The frequency range for the random step is given by the lower and upper limits. The number of evaluation points designates the solution points between each of the extracted modal frequencies within the range (default = 20). The term "bias" sets the spacing of solution points. Increasing the value places the solutions nearer the modal frequencies. The default value is 3 with a value of 1 indicating a uniform spacing. The parameter FREQUENCY SCALE will set the frequency interpolation as linear or logarithmic. A blank or zero value indicates a logarithmic scale and a value of 1 indicates a linear scale.

The random response procedure utilizes modal and global damping. The *MODAL DAMPING and *GLOBAL DAMPING keywords are placed within the *RANDOM RESPONSE analysis step. The *MODAL DAMPING keyword defines the effective damping on a mode-by-mode basis. The mutually exclusive keyword parameters STRUCUTRAL and VISCOUS provide the following modal damping options:

- Direct Modal (VISCOUS = Fraction of Critical Damping)
- Composite Modal (derived from material data, VISCOUS = Composite)
- Rayleigh (alpha-beta damping, VISCOUS = Rayleigh)
- Structural (loss coefficient, STRUCTURAL)

The *MODAL DAMPING keyword parameter DEFINITION designates the manner in which damping values are assigned to the modes. The assignments can be made as a function of mode number or frequency range. Examples of how to assign modal damping are:

```
*MODAL DAMPING, VISCOUS=FRACTION OF CRITICAL DAMPING, DEFINITION =
FREQUENCY RANGE
1.0, 0.04
1200.0, 0.04
2000.0, 0.02

*MODAL DAMPING, STRUCTURAL, DEFINITION = MODE NUMBERS
1, 15, 0.08
16, 20, 0.04
```

The *GLOBAL DAMPING keyword applies to all modes being used in the random response simulation. The total amount of damping that is active in the random response step is obtained by combining modal and global values. The *GLOBAL DAMPING keyword can utilize any combination of the following parameters to define damping:

- ALPHA (mass proportional Rayleigh damping)
- BETA (stiffness proportional Rayleigh damping)
- STRUCTURAL (loss coefficient)

The *PSD-DEFINITION keyword defines the frequency dependent complex-valued scalar functions used in creating the excitation Cross-Spectral Density matrix. It is placed within the model portion of the Abaqus data or at the beginning of input data when performing a random response simulation upon solution restart. Multiple PSD scalar functions can be defined in the model data, but they do not have to be utilized by a random response step. The keyword parameter NAME is required and tags the PSD scalar data for referencing by the *CORRELATION keyword. The *PSD-DEFINITION keyword format is:

*PSD-DEFINITION, NAME= <>
PSD real part, PSD imaginary part, frequency or frequency band number

The data lines contain the real and imaginary PSD values as a function of frequency or the frequency band number if decibel input is utilized. The optional keyword parameters are; TYPE, DB REFERENCE, G, USER and INPUT. TYPE is used to designate the excitation PSD data as relating to base motion, force/moment/pressure, or decibels of excitation relative to a reference value. The parameter DB REFERENCE provides the reference value for decibel input. The G parameter is used when defining an acceleration base motion excitation. The parameter USER activates the UPSD user subroutine, which provides a more general approach for defining the scalar function. The parameter INPUT identifies a supplemental data file for providing the data lines.

With TYPE=BASE the optional parameter G provides a reference gravitational acceleration constant. When G is set to the acceleration due to gravity the PSD values should be defined with units of g^2/Hz, where g is the acceleration due to gravity. The parameter G is used to convert the PSD values into the model's unit of acceleration for solution processing. Any requested acceleration output (PSD or RMS) will be generated with respect to the model's unit of acceleration. The default condition is G=1, which is appropriate when the base motion excitation is defined with a displacement or velocity PSD. When using a displacement or velocity PSD, the base motion Load Case must be defined in a consistent manner, i.e., displacement or velocity.

TYPE= FORCE is used to designate excitations of force/moment/pressure with units of $load^2$/Hz. TYPE=DB provides an alternative format for force/moment/pressure excitations where the input values are given in terms of decibels relative to a reference. The reference value (P_{ref}) is provided via the DB REFERENCE parameter. The data is input at the center band frequencies of full octave bands (f_c). The relationship between the excitation PSD values, $P(f)$ and the data line input, $db(f)$, is:

$$db(f) = 10\log_{10}\frac{P(f)}{\sqrt{2}P_{\text{ref}}/f_{\text{c}}}$$

Band number	Band center (frequency, Hz)
1	1.0
2	2.0
3	4.0
4	8.0
5	16.0
6	31.5
7	63.0
8	125.0
9	250.0
10	500.0
11	1000.0
12	2000.0
13	4000.0
14	8000.0
15	16000.0

As discussed earlier, loads that constitute a random excitation are assigned to a Load Case number when created within the random response step. The keyword options for defining the random loads such as base motion, concentrated loads and connector loads contain a parameter LOAD CASE for this purpose. All distributed loads defined in a random response step are automatically placed into Load Case 1. When distributed loads are present, no other loads may be in Load Case 1. Load Cases are activated within a random response step via the data lines associated with the *CORRELATION keyword. Multiple Load Cases can be activated via one or more instances of the *CORRELATION keyword.

The *CORRELATION keyword has a required parameter PSD, and optional parameters TYPE, INPUT, COMPLEX and USER. The PSD parameter identifies the scalar function that is used with the spatial coupling Cross-Correlation matrices when adding contributions to the nodal load based Cross-Spectral Density matrix. The parameter TYPE is used to indicate if the loads associated with the listed Load Cases are correlated, uncorrelated or represent moving noise. For TYPE=CORRELATED, the loads are statistically dependent, which by default results in unit diagonal and off-diagonal terms in the associated spatial coupling Cross-Correlation matrix. For UNCORRELATED cases only unit (default) diagonal terms are generated. This indicates the loads within the Load Case are statistically independent. With TYPE=CORRELATED or UNCORRELATED each data line consists of a Load Case number along with real and imaginary valued scale factors that are applied to the Load Case's default spatial coupling Cross-Correlation matrix. All Load Cases are independent; therefore, no additional terms of the assembled excitation Cross-Spectral Density matrix are generated for pairs of loads from different Load Cases. For example:

```
*CORRELATION, PSD = PSD-Name, TYPE=CORRELATED
10,   1.2,   0.0
11,   2.4,   1.5
```

All of the load pairs within Load Case 10 and within Load Case 11 are correlated, but load pairs between Load Cases 10 and 11 produce no additional contributions (diagonal or off-diagonal) to the Cross-Spectral Density matrix. The parameter COMPLEX (=YES/NO) designates whether or not both real and imaginary terms are to be included in the Cross-Correlation matrices for the Load Cases listed on the data lines. The parameter INPUT is used to designate an external file which contains the keyword data lines.

By including the parameter USER with the *CORRELATION keyword the UCORR subroutine is activated to generate the Cross-Correlation matrices for the listed Load Cases. UCORR should be utilized when the excitation Cross-Spectral Density matrix is complicated by the presence of partial or weak correlation between excitations. TYPE= CORRELATED or UNCORRELATED is used by the UCORR subroutine to designate how load pairs will be processed. The subroutine can change the default unit values of a Load Case's spatial Cross-Correlation matrix by defining weighting factors that override the scaling factors listed in the *CORRELATION data lines.

The nodal load based Cross-Spectral Density matrix for TYPE=MOVING NOISE is dependent on the relative position of the points where the loads are applied. Therefore, the moving noise option can only be used with concentrated and distributed loads. Also, the excitation Cross-Spectral Density matrix must be real-valued, requiring the PSD scalar function be real-valued (no imaginary terms). Moving noise correlation allows only a single *CORRELATION keyword to be present in the *RANDOM RESPONSE step. Also, the COMPLEX and USER keyword parameters cannot be used with TYPE=MOVING NOISE. The data line for a moving noise random simulation consists of a Load Case number followed by the global vector components of the noise velocity and the name of a PSD scalar function:

```
*CORRELATION, TYPE=MOVING NOISE
Load Case #, V_X, V_Y, V_Z, PSD NAME
```

2.2. Abaqus Subroutine UCORR

Prior to a more detail discussion of the behaviors associated with the UCORR subroutine, it is beneficial to review a few key items.

- The *CORRELATION keyword is used to assemble the excitation nodal load based Cross-Spectral Density matrix. It references a PSD scalar function and designates the loads that constitute individual Load Cases as being correlated or uncorrelated.
- The effect of multiple *CORRELATION keywords within a Random Response step is additive when assembling the nodal load based Cross-Spectral Density matrix.
- Load Case contributions to the nodal load based Cross-Spectral Density matrix are obtained from the product of a complex frequency dependent PSD scalar function and the Load Case's spatial coupling Cross-Correlation matrix.
- Correlated loads produce diagonal and off-diagonal (i.e., coupling) terms of a Load Case's Cross-Correlation matrix that are unity by default.
- Uncorrelated loads always produce off-diagonal terms of a Load Case's Cross-Correlation matrix that are zero. By default, the diagonal terms of the Cross-Correlation matrix are unity for uncorrelated loads.
- The data lines of the *CORRELATION keyword provide real and imaginary scale factors that are used as Cross-Correlation matrix weighting factors.
- For a given *CORRELATION instance, the UCORR subroutine is activated by the keyword parameter USER. The UCORR subroutine is used to define weight factors to be applied to Cross-Correlation matrices, which can be applied to both diagonal and off-diagonal terms. When UCORR is utilized the scaling factors on the *CORRELATION data lines are ignored.

The UCORR subroutine is called multiple times for each instance of a *CORRELATION keyword in which it is used. Subroutine calls are made for all degrees-of-freedom of the nodes to which loads have been applied within the step, not just to the degrees-of-freedom that correspond to the loads within the Load Case being processed. However, only the returned weighting factors that are associated with the Load Case being processed are utilized when updating the Cross-Spectral Density matrix. The UCORR Subroutine has the following FORTRAN format;

```
SUBROUTINE UCORR (PSD,CORRR,CORRI,KSTEP,LCASE,JNODE1,JDOF1,
1 JNODE2,JDOF2,COOR1,COOR2)
```

Passed In:	PSD	PSD scalar function
	KSTEP	Simulation Step Number
	LCASE	Load Case Number (Internal)
	JNODE1	First node of a load pair
	JDOF1	Degree of Freedom for load at JNODE1
	JNODE2	Second node of a load pair
	JDOF2	Degree of Freedom for load at JNODE2
	COORD1	Array of JNODE1 coordinates
	COORD2	Array of JNODE2 coordinates
Returned:	CORRR	Real part of Cross-Correlation weighting factor
	CORRI	Imaginary part of Cross-Correlation weighting factor

The returned variable CORRR is the real component of the weighting factor and CORRI is the imaginary component. When the designation TYPE=CORRELATED is used with the UCORR subroutine the returned output will include terms associated with degree-of-freedom pairs from individual and different nodes. Diagonal and off-diagonal terms associated with these degree-of-freedom pairs are used in updating the Cross-Correlation matrix. When the designation TYPE=UNCORRELATED is used, the returned output will not include off-diagonal terms associated with degree-of-freedom pairs from different nodes. Diagonal and off-diagonal terms associated with degree-of-freedom pairs from individual nodes are returned. However, the off-diagonal terms are not utilized when updating the Cross-Correlation matrix due to the TYPE=UNCORRELATED designation.

The variable LCASE that is passed into the UCORR subroutine refers to the internal Load Case number and not to the user defined Load Case number. The user defined Load Case numbers are compressed to generate internal numbers that are consecutive starting at 1. Load Case and Correlation internal data can be viewed by generating a preprint of the simulation history data (*PREPRINT, HISTORY=YES). The variables JNODE1 and JNODE2 passed into the UCORR subroutine refer to internal node numbers. The internal node numbers are available by generating a preprint of the basic model data (*PREPRINT, MODEL=YES). Data generated by the UCORR subroutine can be written to the output file (.dat) by writing to unit 6 or the message file (.msg) by writing to unit 7. You do not have to open unit 6 or unit 7 in the subroutine because they are opened automatically by Abaqus. The UCORR subroutine is demonstrated and discussed further in Section 3.3.

3. Cantilever Beam Example

The cantilever beam shown in Figure 3 is excited by vertical forces at two locations. The desired output is the vertical velocity response at the beam's tip. The beam is an aluminum rod with a length of 0.609 meters (24 inches) and a diameter of 0.0127 meters (0.50 inches). Excitation forces are applied at the ¼-span and mid-span locations as shown in Figure 3. The *PSD-DEFINITION scalar function is the same for both force locations and is real-valued at 1.0 N^2/Hz over the frequency range of 5 to 2500 Hz. The root mean square force corresponding to the excitation PSD is 49.95 N-rms. Modal damping of 2% critical was assigned to all modes within the frequency range.

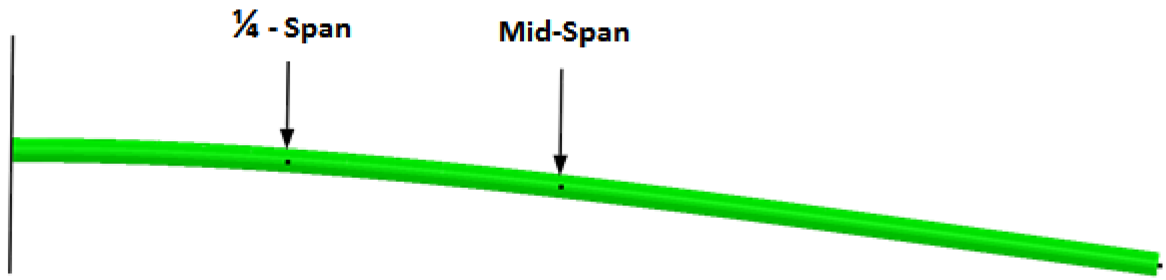


Figure 3: Cantilever Beam Illustrative Example

3.1. Uncorrelated Excitations

The following random response simulations were performed for uncorrelated excitations:

Simulation 1: Force excitation is only at the ¼-span location

Simulation 2: Force excitation is only at the mid-span location

Simulation 3: Uncorrelated excitations at ¼-span and mid-span locations

The cantilever beam has a total of six vertical bending modes below 2500 Hz. Figure 4 shows the six mode shapes with the excitation locations highlighted in red. For bending modes 1 and 2 the motion of the excitation points are in-phase, that is, they have the same sign (+/-) in the eigenvector. The excitation points are out-of-phase for modes 4 and 6, with the magnitudes at each location being nearly the same for Mode 4. Modes 3 and 5 have nearly no vertical motion at the mid-point location.

The model data input used to define the PSD scalar function is:

```
*PSD-DEFINITION, NAME=PSD-DEF
```

1.0, 0.0, 1.0
1.0, 0.0, 2500.0

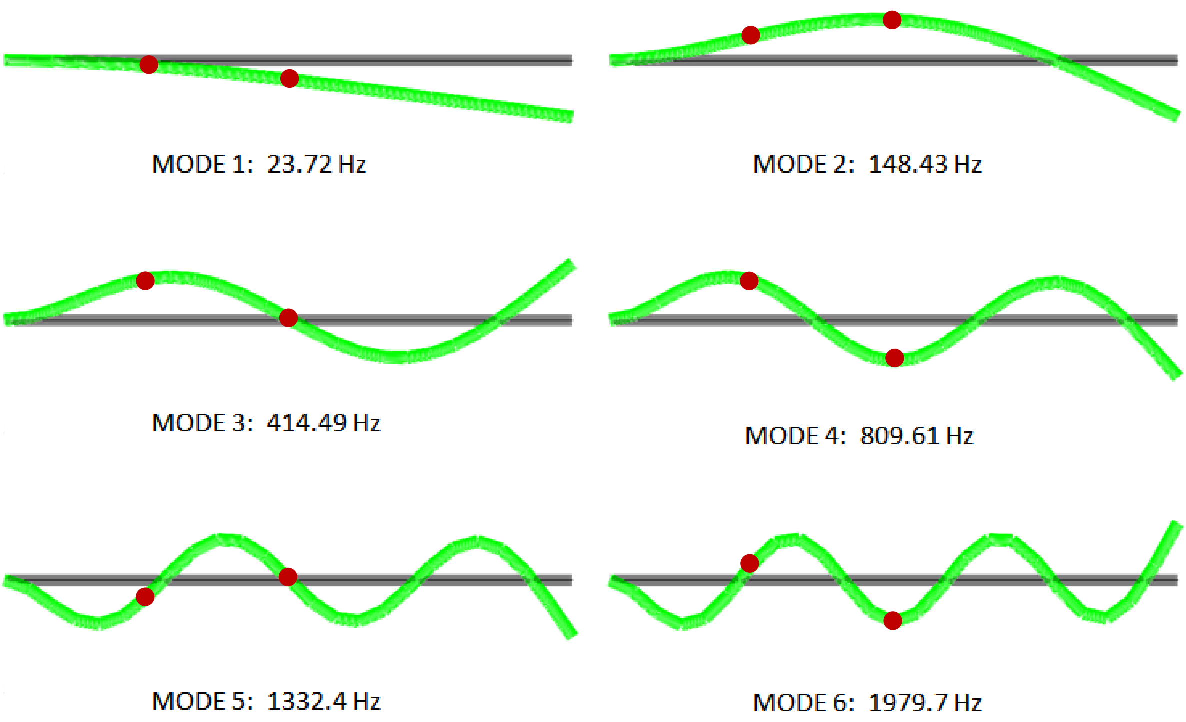


Figure 4: Cantilever Beam Bending Modes

Simulations 1 and 2 have only a single point of excitation so the TYPE parameter (default is CORRELATED) could be omitted from the *CORRELATION keyword. The excitations are defined as unit vertical forces and are assigned to Load Case 1. The Load Case 1 designation can be used since there is no pressure loading defined. Load Case 1 is activated via the data line associated with the *CORRELATION keyword. The frequency range and number of solution frequencies for determining the output PSDs are provided on the data line of the *RANDOM RESPONSE keyword. Mode selection and modal damping are also included under the overall step definition. All extracted modes were utilized so the modal selection keyword was not required. The keyword data for Simulation 1 is:

```
*STEP, NAME=RANDOM, PERTURBATION
Simulation #1:  Single Excitation at Quarter-Span Location
*RANDOM RESPONSE
5.0,  2500.0,  40,  1.,
*MODAL DAMPING, DEFINITION=FREQUENCY RANGE
1.0,          0.02
2500.0,       0.02
```

```
*CLOAD, LOAD CASE = 1
QTR-SPAN,  2,  1.0
*CORRELATION, PSD=PSD-DEF, TYPE=UNCORRELATED
1, 1.0
```

History output at each solution frequency was requested for motions at the ¼-span, mid-span and free-end locations. The output consisted of PSD and RMS values for vertical displacement, velocity and acceleration. History output was also requested for the modal generalized displacement, velocity and acceleration PSDs. Full field output of nodal displacement, velocity and acceleration plus element stress tensor components was requested for every 5th solution frequency. Requests for significant amounts of field output can require generating a very large number of response PSDs, with associated increase in computer memory requirements and processor usage. To reduce the computational cost of a random response step, you can request output be written for selected element and node sets. Abaqus will calculate the response for only the element and nodal variables requested. Note; a request for MISES or RMISES field output requires that stress tensor data be output with the eigenvectors in the frequency extraction step. The Abaqus keywords and data lines for requesting output in the simulations are:

```
*OUTPUT, FIELD, FREQUENCY=5
*NODE OUTPUT
AT, UT, VT, RA, RU, RV
*ELEMENT OUTPUT, DIRECTIONS=YES
RS, S
*OUTPUT, HISTORY, FREQUENCY=1
*MODAL OUTPUT
GA, GU, GV
*NODE OUTPUT, NSET=FREE-END
A2, RA2, RU2, RV2, U2, V2
*NODE OUTPUT, NSET=MID-SPAN
A2, RA2, RU2, RV2, U2, V2
*NODE OUTPUT, NSET=QTR-SPAN
A2, RA2, RU2, RV2, U2, V2
```

Simulation 3 has UNCORRELATED excitations at the ¼-span and mid-span locations. This condition can be attained via two different methods. The first method is to assign both loads to Load Case 1 and set TYPE= UNCORRELATED on the *CORRELATION keyword. The second method is to define the loads with separate Load Case designations. Activating the two load cases under different or the same *CORRELATION keyword would automatically produce an uncorrelated response (recall that no correlation can occur between separate Load Cases). The two methods are shown below.

```
*CLOAD, LOAD CASE = 1
QTR-SPAN,   2, 1.0
MID-SPAN,  2,  1.0
*CORRELATION, PSD=PSD-DEF, TYPE=UNCORRELATED
1, 1.0
*CLOAD, LOAD CASE = 1
QTR-SPAN,   2, 1.0
*CLOAD, LOAD CASE = 2
MID-SPAN,  2,  1.0
*CORRELATION, PSD=PSD-DEF
1, 1.0
2, 1.0
```

Figure 5 shows the Power Spectral Density free-end velocity response curves obtained from the history output of Simulations 1, 2 and 3. The PSD curve of Simulation 3 is equal to the sum of the PSD curves from Simulations 1 and 2. Since the uncorrelated excitations in Simulation 3 are processed as statistically independent with no cross-correlation, that is, no off-diagonal terms in the Cross-Spectral Density matrix, their combined effect is additive.

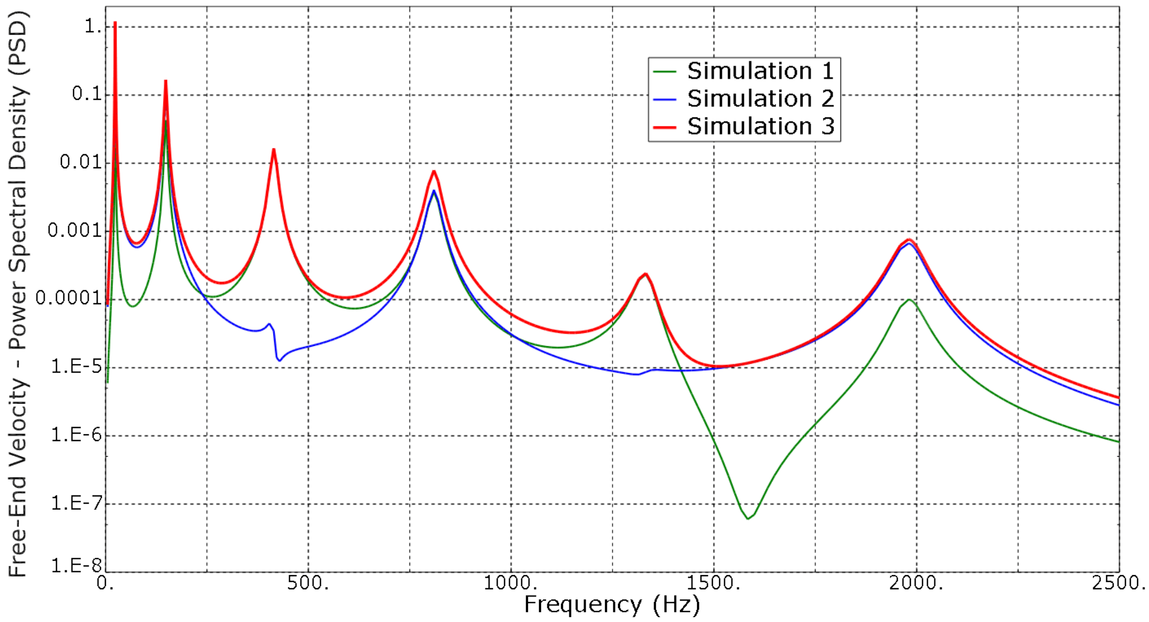


Figure 5: Free-End Velocity PSD Curves

The “Mean Square” response can be obtained by integrating the response PSD curves. Just as with the response PSDs, the Mean Square output for Simulation 3 is equal to the sum of that for Simulations 1 and 2 (Figure 6). Note that the Mean Square values are also referred to as the Variance of the output. Notice in Figures 5 and 6, the local peaks in the

PSD curves and the abrupt increases in the Variance curves coincide with the resonant frequencies. Taking the square root of the Variance curve will produce a RMS response curve. The RMS value at any given frequency only contains the response contributions from the excitation frequencies below that frequency. The square root of the final data point on the Variance curve represents the RMS value over the entire frequency range.

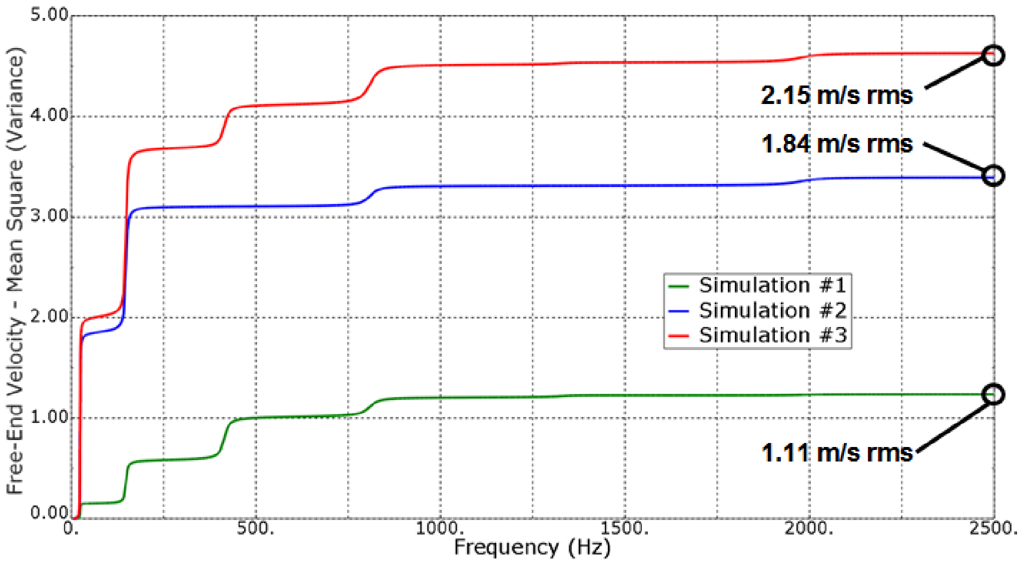


Figure 6: Free-End Velocity Mean Square (Variance) Curves

3.2. Correlated Excitations

A fourth simulation utilizes correlated excitations at the ¼-span and mid-span locations. The random response input for this situation is:

```
*CLOAD, LOAD CASE = 1
QTR-SPAN, 2, 1.0
MID-SPAN, 2, 1.0
*CORRELATION, PSD=PSD-DEF, TYPE=CORRELATED
1, 1.0, 0.0
```

For Simulation 4 the Cross-Correlation matrix associated with the excitation Load Case includes both diagonal and off-diagonal terms. Therefore, the excitation Cross-Spectral Density matrix assembled from the PSD scalar function and the Cross-Correlation matrix also includes off-diagonal terms. The transfer functions in conjunction with the Cross-Spectral Density off-diagonal terms accounts for the phasing (sign), location and magnitudes of the applied correlated loads. Figure 7 and 8 shows the free-end velocity response PSD and Variance curves for Simulations 3 and 4. Notice the significant drop in the Simulation 4 PSD associated with the response of Mode 4 near 810 Hz. This behavior is the result of the excitation forces at the quarter-span and mid-span locations being correlated (dependent). They act with the same initial phasing (sign) over all times and frequencies. As a result, Mode 4 is not being excited due to one force acting in-phase and the other acting out-of-phase with displacements of the mode shape that have approximately the same magnitude. The Variance curves indicate the random response contributions associated with modes 1 and 2 are significantly higher for the correlated loads (Simulation 4) as compared to the uncorrelated case (Simulation 3), while the correlated response associated with Mode 4 is nearly zero. The Variance curves illustrate why Simulation 4 has the higher RMS response.

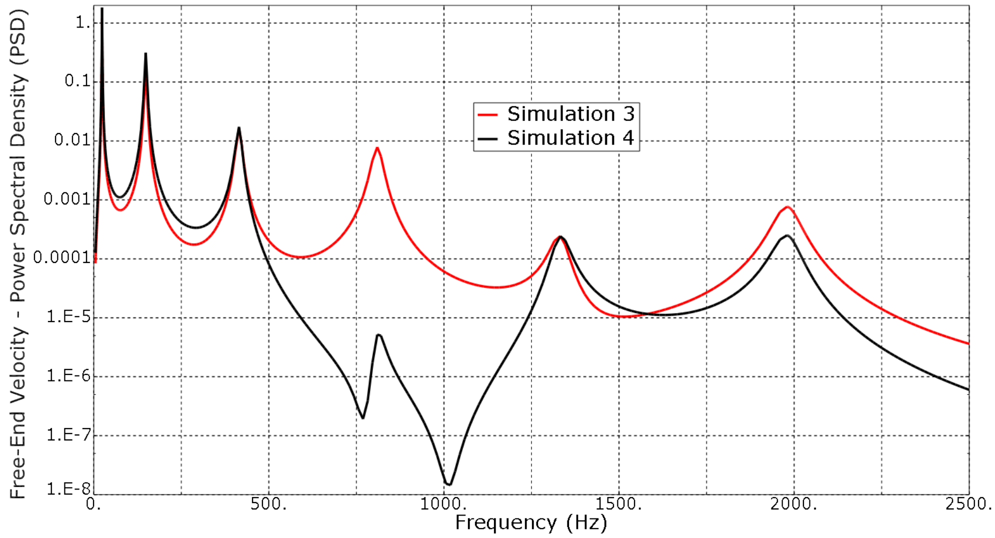


Figure 7: Free-End Velocity PSD Curves (Simulations 3 and 4)

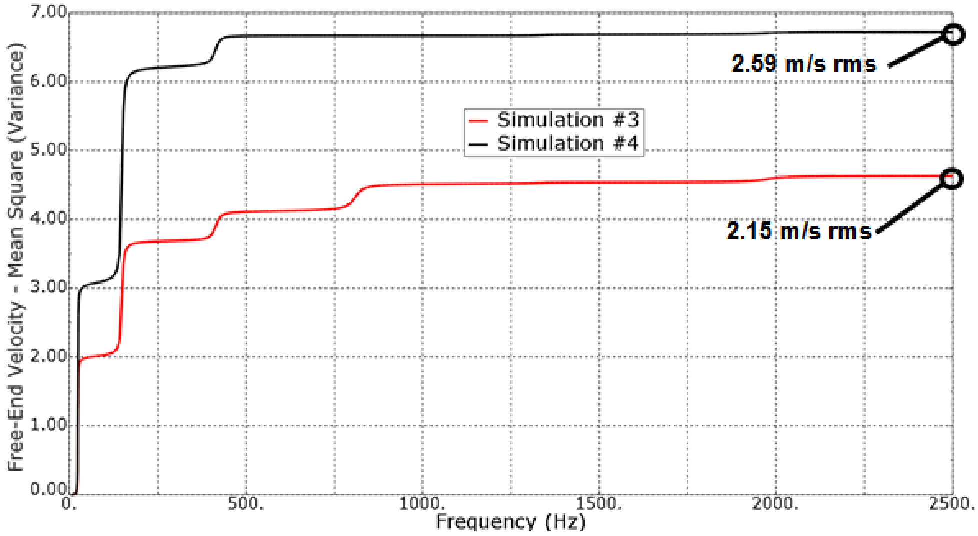


Figure 8: Free-End Velocity Mean Square (Variance) Curves (Simulations 3 and 4)

Contour plots of the Root Mean Square bending stress (component S33 in this case) for Simulations 3 and 4 are provided in Figure 9. The S33-rms values were obtained from the last frame of field data on the simulation output database.

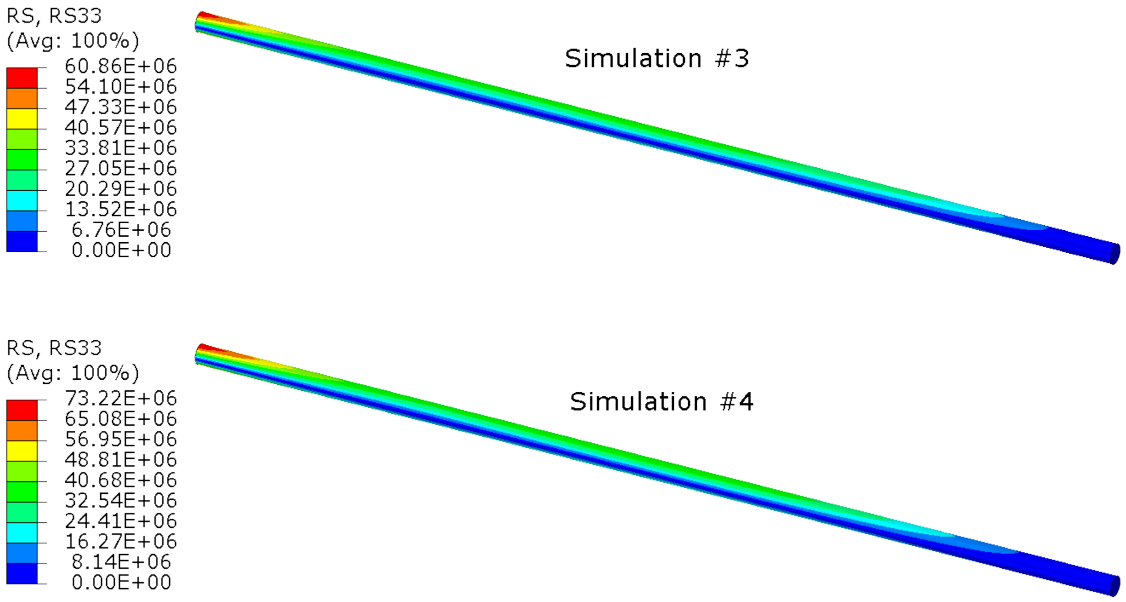


Figure 9: Beam S33-rms Bending Stresses for Simulations 3 and 4

3.3. UCORR Subroutine Example

An example of the UCORR subroutine uses the previous cantilever beam model with an additional load point at the ¾ span positon. A baseline simulation was performed consisting of all three forces, each of unit magnitude, and assigned to the Load Case 10. The designation TYPE=CORRELATED is used to create a fully populated 3x3 excitation Cross-Spectral Density matrix. A second simulation utilizes multiple *CORRELATON instances and the UCORR subroutine to generate the identical 3x3 Cross-Spectral Density matrix.

The beam model identifies nodes at ¼, ½ and ¾ span locations as FORCE-1, FORCE-2 and FORCE-3, respectively. The random response keyword data input for the baseline simulation is:

```
*PSD-DEFINITION, NAME=PSD-DEF
1.0, 0., 1.
1.0, 0., 2500.

*STEP, NAME=BASELINE, PERTURBATION
BASELINE BEAM:  Three Forces (Correlated)
*RANDOM RESPONSE
```

```
5.0, 2500.0, 40, 1.,
*MODAL DAMPING, DEFINITION=FREQUENCY RANGE
1.0,      0.02
2500.0, 0.02
*CLOAD, LOAD CASE = 10
FORCE-1,  2, 1.
FORCE-2,  2, 1.
FORCE-3,  2, 1.
*CORRELATION, PSD=PSD-DEF, TYPE=CORRELATED
10, 1.0, 0.0
```

The second simulation, designated as UCORR-SIM, uses individual Load Cases for each of the three forces to create the diagonal terms of the Cross-Spectral Density matrix. These Load Cases are given the user defined values 11, 22, 33 corresponding to forces at nodes FORCE-1, FORCE-2 and FORCE-3, respectively. The coupling off-diagonal terms are created with the UCORR subroutine via Load Cases defined for each force excitation pair. These Load Cases are given the user defined values 12, 13, 23 corresponding to force pairs (FORCE-1, FORCE-2), (FORCE-1, FORCE-3) and (FORCE-2, FORCE-3), respectively. The subroutine written for UCORR-SIM returns CORRI = 0.0 (imaginary weight factor) for all nodal load combinations. The subroutine sets CORRR=0.0 (real weight factor) as a default, while generating a value of 1.0 when JNODE1 does not equal JNODE2. The components of the Cross-Correlation matrix derived from calls to the UCORR subroutine will have zeros for diagonal terms and unit values for off-diagonal terms associated with the various load pairs. The assembly of the Cross-Spectral Density matrix from all of the *CORRELATION instances will be identical to that of the baseline simulation. The subroutine used in this example and the associated random response keywords for the UCORR-SIM analysis are:

```
      SUBROUTINE UCORR(PSD,CORRR,CORRI,KSTEP,LCASE,JNODE1,JDOF1,
     1 JNODE2,JDOF2,COOR1,COOR2)
C
      INCLUDE 'ABA_PARAM.INC'
C
      DIMENSION COOR1(3),COOR2(3)
      CHARACTER*80 PSD
C
      CORRI=0.0
      CORRR=0.0
      IF(JNODE1.NE.JNODE2) CORRR=1.0
      WRITE(6,*) PSD, KSTEP, LCASE, JNODE1, JDOF1, JNODE2, JDOF2, CORRR, CORRI
      RETURN
      END
```

```
*CLOAD, LOAD CASE = 11
FORCE-1, 2, 1.
*CLOAD, LOAD CASE = 22
FORCE-2, 2, 1.
*CLOAD, LOAD CASE = 33
FORCE-3, 2, 1.
```

```
*CLOAD, LOAD CASE = 12
FORCE-1, 2, 1.
FORCE-2, 2, 1.
*CLOAD, LOAD CASE = 13
FORCE-1, 2, 1.
FORCE-3, 2, 1.
*CLOAD, LOAD CASE = 23
FORCE-2, 2, 1.
FORCE-3, 2, 1.
**
*CORRELATION, PSD=PSD-DEF, TYPE=UNCORRELATED
11, 1.0, 0.0
*CORRELATION, PSD=PSD-DEF, TYPE=UNCORRELATED
22, 1.0, 0.0
*CORRELATION, PSD=PSD-DEF, TYPE=UNCORRELATED
33, 1.0, 0.0
**
*CORRELATION, PSD=PSD-DEF, TYPE=CORRELATED, USER
12, 1.0, 0.0
*CORRELATION, PSD=PSD-DEF, TYPE=CORRELATED, USER
13, 1.0, 0.0
*CORRELATION, PSD=PSD-DEF, TYPE=CORRELATED, USER
23, 1.0, 0.0
```

Note: a different PSD scalar function could have been defined for each *CORRELATION instance, thereby providing the analyst sufficient flexibility to input a general Cross-Spectral Density matrix with varying degrees of correlation between excitations. The output PSD for the vertical velocity at the tip of the beam is shown in Figure 10, with the two simulations producing identical results.

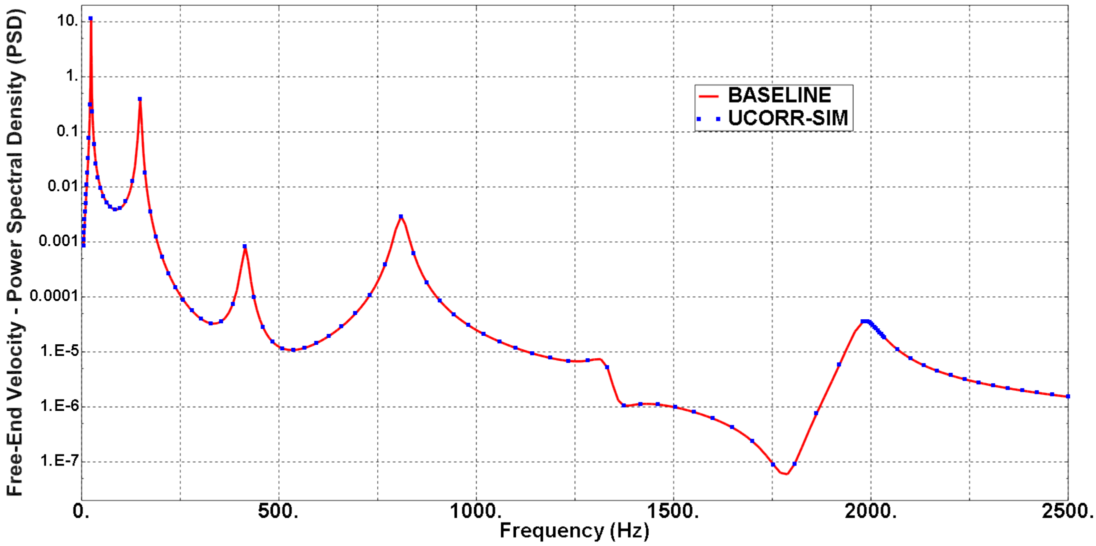


Figure 10: Free-End Velocity PSD Curves (UCORR Example)

4. Random Response Simulation with Abaqus/CAE

The *RANDOM RESPONSE keyword is fully supported within the Abaqus/CAE step module. As shown in Figure 11, the random response step must follow a frequency extraction step.

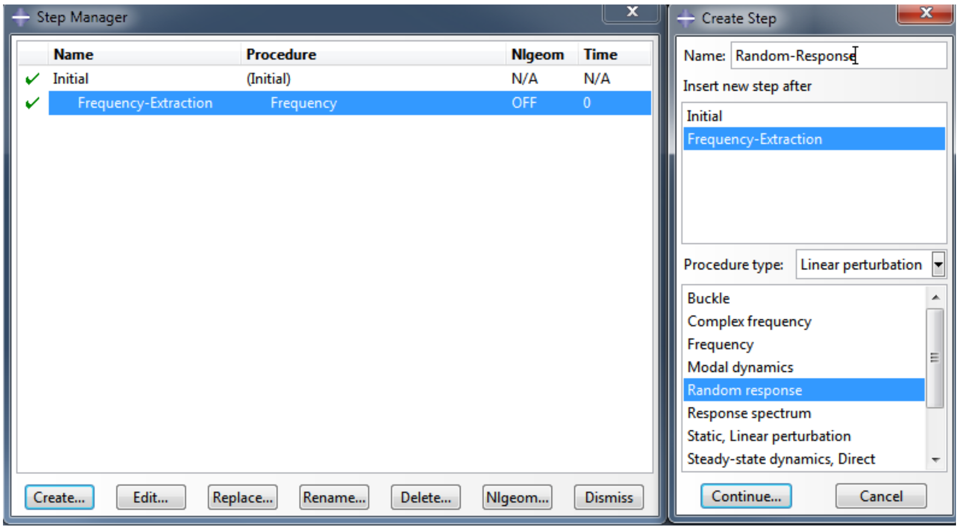


Figure 11: Creating a Random Response Step with Abaqus/CAE

The **Edit Step** dialog box **Basic** tab provides for the input of all required data associated with the *RANDOM RESPONSE keyword. The **Damping** tab allows the definition of any of the four types of modal damping, based upon either mode number or frequency ranges. Examples of the **Basic** and **Damping** tabs are shown in Figure 12. Abaqus/CAE does not support global damping. However, the *GLOBAL DAMPING keyword can be included in an Abaqus/CAE model by means of the keyword edit procedures.

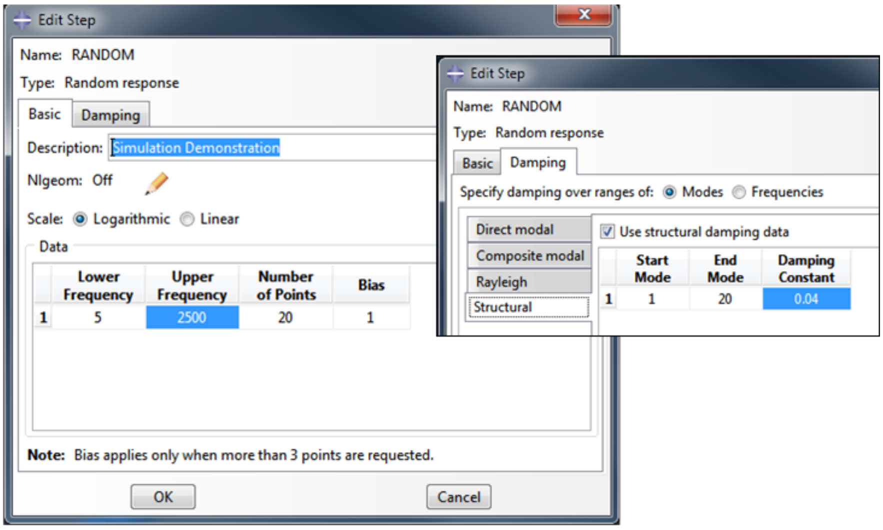


Figure 12: Random Response Step Dialog Box

Output requests associated with random response simulations are fully supported by Abaqus/CAE, as illustrated in Figure 13 for field output and Figure 14 for history output. The standatd variable identifiers represent PSD values while the identifiers that are preceded by 'R', represent RMS values with the frequency integration performed up to the solution frame being written to the output database.

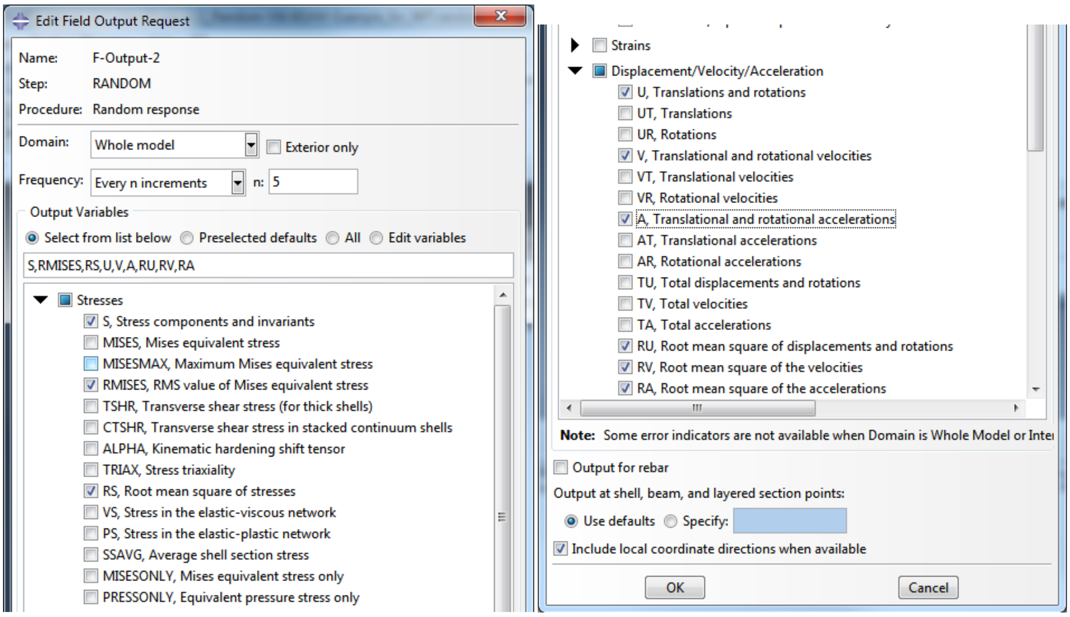


Figure 13 : Example Field Output Request for a Random Response Simulation

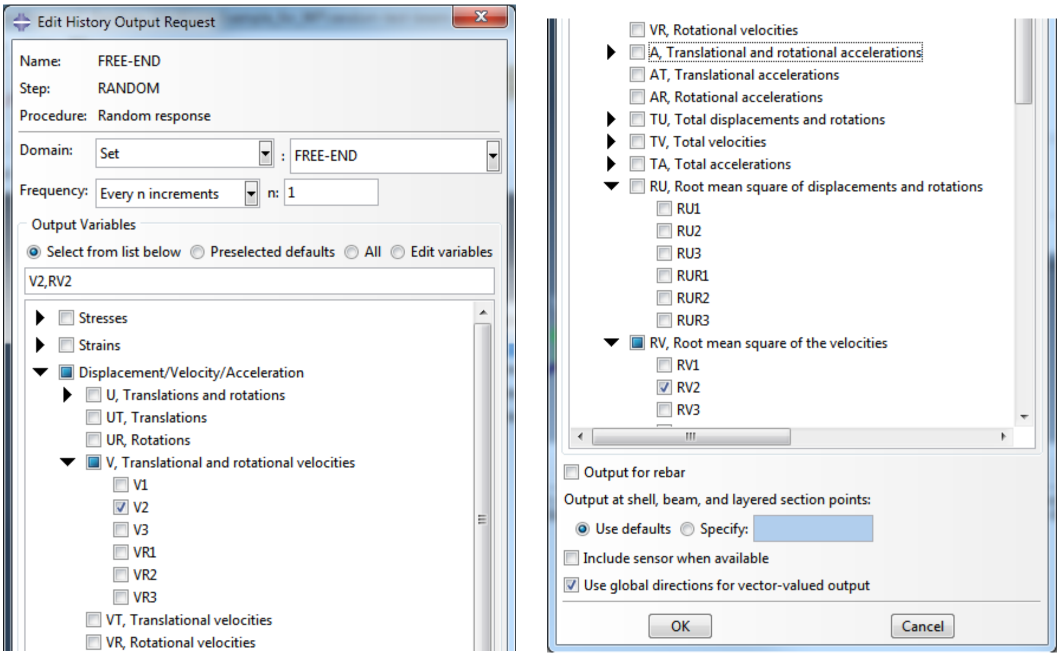


Figure 14 : Example History Output Request for a Random Response Simulation

Abaqus/CAE supports the *PSD DEFINITION keyword via the amplitude creation capability (**Tools** > **Amplitude** > **Create**). The amplitude type is set to **PSD Definition** and three options are provided; Power, Decibel, and Gravity (base motion). The **Power** and **Gravity** options allow the user to **Specify data in an external user subroutine**. Figure 15 illustrates the **Abaqus/CAE** dialog box formats that are used to define the PSD scalar functions. Note; when the **Specification units** are set to **Decibel**, the data column labeled as **Frequency** is used to input the frequency band number. This is detailed in the Abaqus documentation under **Abaqus** > **Analysis** > **Analysis Procedures** > **Dynamic stress/displacement analysis** > **Random response analysis** > **Defining the frequency functions**.

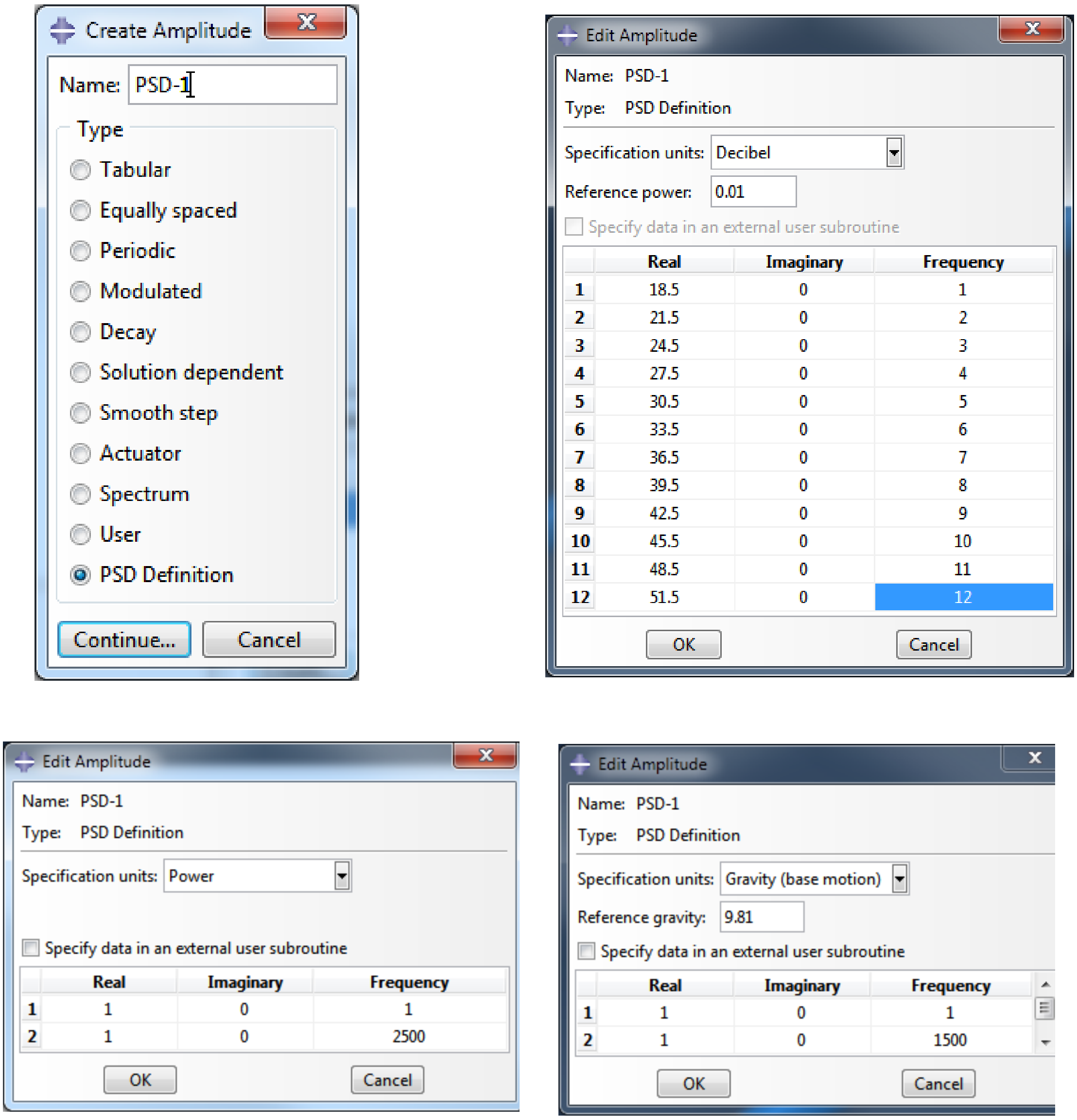
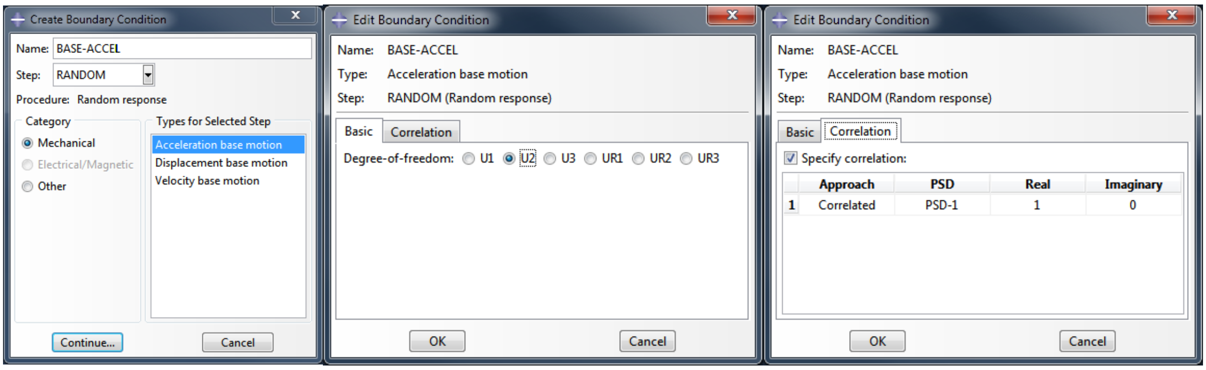


Figure 15: Definition of a Scalar PSD Function

Abaqus/CAE does not support the definition of random excitations as nodal forces/moments, connector forces or pressure loads. These load types must be implemented via keyword editing or direct editing of the input file once it has been generated. These load types must include the LOAD CASE parameter when used within a random response step. The *CORRELATION keywords needed to activate these load types must also be generated via keyword or input file edits.

The only random excitation type supported by Abaqus/CAE is base motion. The CAE interface allows to create a boundary condition within the random response step that can be used to define displacement, velocity, or acceleration base motions. The **Edit Boundary Condition** dialog box is used to define the direction of the excitation via one of the six global degrees of freedom. The **Correlation** tab is used to activate the *CORRELATION keyword when generating the input file. Figure 16 demonstrates this process for base motion acceleration in the global U2 direction. The SCALE and AMPLITUDE parameters on the *BASE MOTION keyword are always ignored in a random response step, since the unit motions (displacement, velocity or acceleration) are utilized. CAE will allow multiple base motion definitions within a random step, but each is defined as a separate Load case with separate *CORRELATION keywords. Multiple base motions can be combined within the same Load Case via the keyword EDITOR, but the base motion in each direction is always set to unity. Therefore, defining multiple correlated base motions within a single Load Case may require to apply adjustments to the excitation scalar PSD function. An alternative approach for correlated base motions would be to reposition the model in the global coordinate system so that the resultant base motion is aligned with one of the global axes.




```
*BASE MOTION, TYPE=ACCELERATION, DOF=2, SCALE=1.0, LOAD CASE=1
*CORRELATION, TYPE=CORRELATED, PSD=PSD-1, COMPLEX=NO
1, 1.0, 0.0
```

Figure 16: Base Motion Acceleration Excitation

5. Random Response of Fluid-Filled Structures

The random response of fluid-filled structures, such as an oil pan on a diesel engine can be simulated with Abaqus. However, it requires a modeling approximation for the entrained fluid in which displacement based elements are utilized along with contact modeling between the fluid and structures. You can also refer to this information in the Abaqus Knowledge Base QA article, QA00000048796.

To include the added-mass influence of contained fluid in an Abaqus Random Response linear perturbation analysis, the fluid must be modeled with displacement based continuum elements. The fluid behavior is approximated as being inviscid and nearly incompressible. To attain this approximation, the fluid is meshed with C3D8H, C3D8RH or C3D4H elements using the Neo-Hookean hyperelastic material model. The bulk modulus (K) can be set to that of the fluid via the material constant value D = 2/K. The shear modulus (G) is input with the material constant C10 = G/2. The shear modulus must be set to a sufficiently low value such that all deformation modes associated with material shearing are shifted to frequencies below the minimum frequency being used in the random response simulation. In most cases this means the shear modulus is set near zero for the fluid approximation. The fluid model does not include any direct representation of viscosity (inviscid assumption), but the influence of the fluid on the damping behavior of the system can be indirectly assigned via modal techniques (illustrated later). Likewise, the hourglass stiffness for C3D8RH elements must be defined at a sufficiently low value such that the frequencies associated with hourglass modes are below the minimum frequency of interest.

Hybrid element formulations are used for the fluid mesh due to the large difference between the shear modulus (near zero) and the bulk modulus. However, even when using the hybrid formulation round-off errors associated with the large mismatch between bulk and shear can produce eigenvalues characterized by deformation modes that appear to be similar to hour-glassing. A plot of the pressure stress associated with these deformation modes indicate they are actually triggered by volume locking behavior. These volume locking modes are extremely difficult to excite; and therefore, have little influence on any subsequent Steady State or Random Response simulation. When the expected acoustic wavelengths in the fluid are long compared to the structural dimensions the presence of volume locking induced modes can be further reduced by artificially increasing the bulk modulus. The increased bulk modulus can shift some of the volume locking deformation modes above the maximum frequency of interest for the random response simulation. Artificially increasing the bulk modulus (reducing D) provides a solution that more closely approximates classical incompressible “added-mass” behavior. It is recommended that some small value of D be used rather than a zero value.

The interface between the fluid and structure is modeled with contact. The wetted surface of the structure should be the master surface with the interfacing fluid, the slave surface. The contact properties should have the following characteristics:

- Hard Contact (Direct – Lagrange Multiplier)
- No Separation
- Small Sliding (Node to Surface formulation is preferred)
- Frictionless
- Initial Clearance set to 0.0 (Initial contact state is closed)

The small sliding formulation is used with initial clearances set to zero (all slave nodes are closed). The initial contact status is utilized as the current state for the linear perturbation frequency step that must precede the Random Response step. The Random Response step is very similar to mode-based Steady State Dynamics, in that it creates the modal transfer functions that are then used in the random response calculations. The fluid modeling technique described herein is valid for linear (small displacement) frequency domain simulations, but may not be appropriate for some direct transient dynamic simulations. The presence of a significant preload condition may require the definition of initial stress conditions or the use of *IMPORT procedures.

EXAMPLE : Oil Filled Pan

Pan (Steel):

- 12" wide x 18" long x 4" deep (0.3048m x 0.4572m x 0.1016 m)
- Bottom edge fillet radius is 1" (0.0254m)
- Built-in support along the upper perimeter
- Wall Thickness = 0.0625" (0.0015875m)
- Element type = S4R
- Steel: E = 30e6 psi = 206.84e9 Pa, Poisson's Ratio = 0.33
- Specific Weight = 7.83
- Mass Density = 0.000732 Lb-sec^2/in^4 = 7830 Kg/m^3
- Composite Damping Ratio (mass weighted) = 0.05

Oil:

- 2.5" Deep (0.0635m)
- Element Types: C3D8RH, C3D8H, C3D4H
- Specific Weight = 0.85
- Mass Density = 0.0000793 Lb-sec^2/in^4 = 793 Kg/m^3
- C10 = 1.0e-6 (Shear Modulus = 2.0e-6 psi = 0.0138 Pa)
- D = 5.382e-6 (Bulk Modulus = 371610 psi = 2562.2e6 Pa)
- D = 1.0e-8 (alternate high bulk modulus of 200e6 psi = 1.38e12 Pa)
- Composite Damping Ratio (mass weighted) = 1e-8 (~ inviscid)
- Alternate Composite Damping Ratio (mass weighted) = 0.02

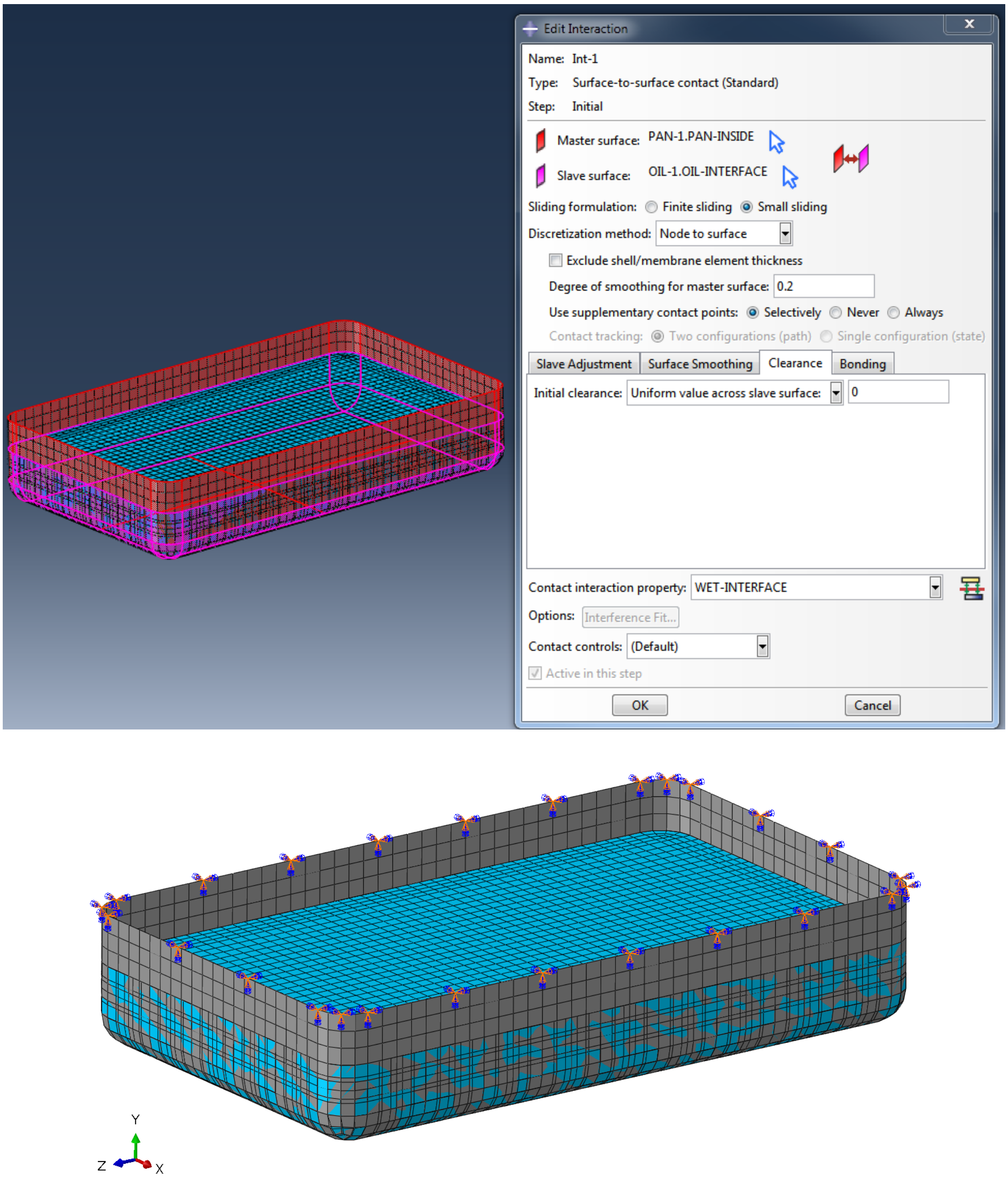
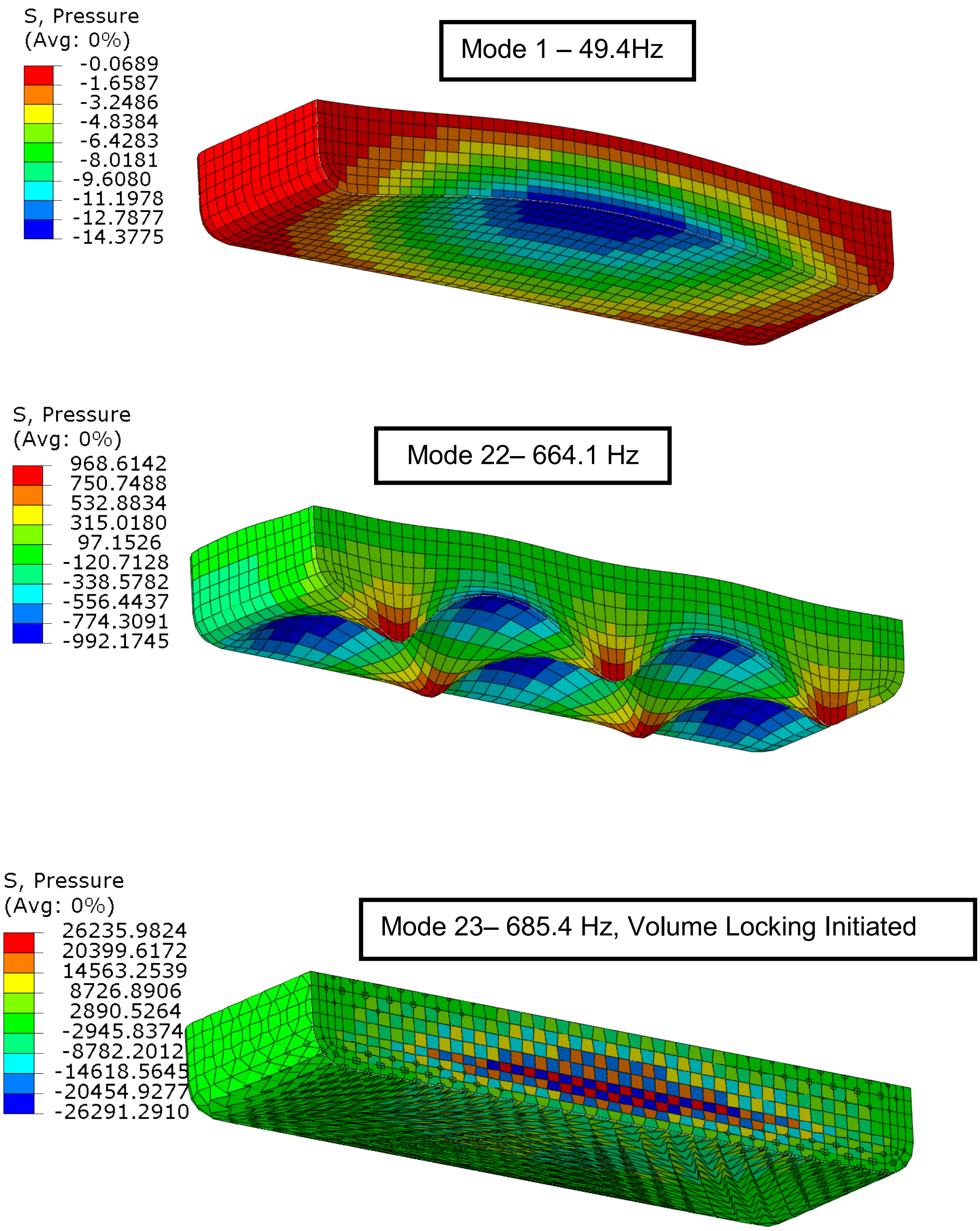


Figure 17: Oil Filled Pan Example



- C3D8RH oil model with the actual bulk modulus is shown.
- Mode shapes via a view cut through the pan center.
- Pan is not shown.
- Contour plots of fluid pressure stress.

Figure 18: Frequency Extraction Results

The added mass influence of an inviscid fluid on a structure's response under small amplitude vibrations can also be modeled with an acoustic representation of the fluid. This approach can be used for steady state dynamics but is not available for use with the random response procedure. However, since the random response procedure utilizes a modal technique, a validation of the displacement based fluid modeling approach can be made by comparing steady state dynamic solutions obtained from both acoustic and displacement based fluid models. The dynamic excitation for performing this comparison is a unit pressure applied to the entire outside surface of the pan. The acoustic based model provides coupled structural-acoustic modes, and both models perform modal steady state dynamic simulations with the traditional Abaqus architecture. Composite modal damping (mass basis) was implemented with a pan material damping ratio of 0.05 and the oil damping ratio set near zero at 1.0e-8. Shown in Figure 19 is the frequency response function for the velocity magnitude at the bottom center of the pan.

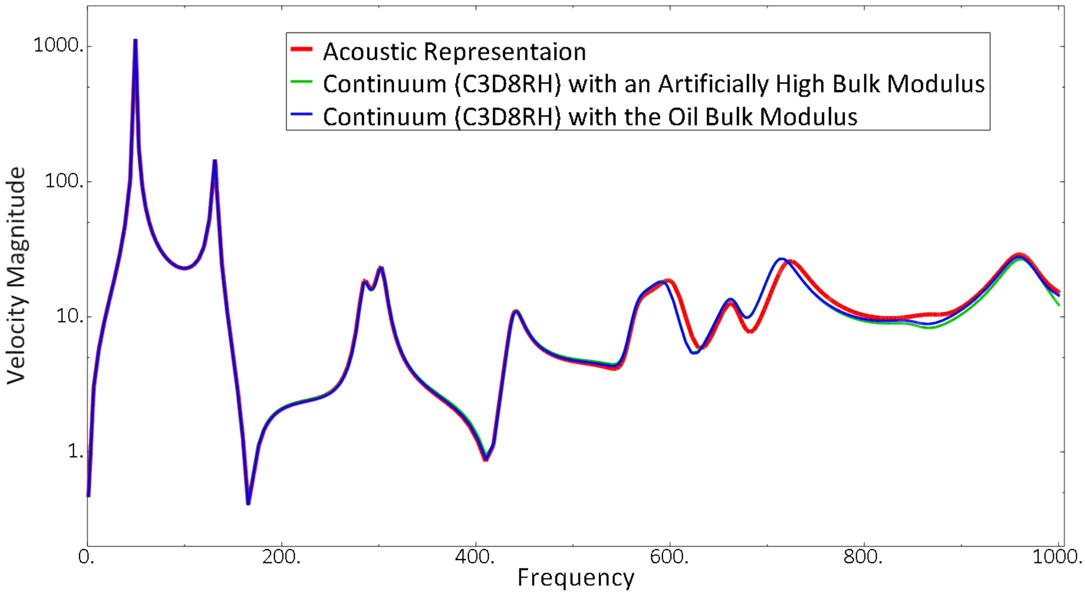


Figure 19: Oil Filled Pan Steady Sate Dynamic Results

Differences between the acoustic and continuum representation of the oil are observed at the higher frequencies (> 500Hz). The differences are related to mesh refinement. The acoustic elements have a continuous piecewise linear interpolation of pressure and the continuum elements have a piecewise constant representation. There is no significant influence of the volume locking modes (#23 and #37) on the solution; as illustrated by the high bulk modulus case in which these two modes were not present. The excellent correlation between the acoustic and displacement based fluid representations illustrates that the displacement method can indeed be used to represent added mass behavior in steady state dynamics, and therefore, random response simulations.

The displacement based oil model was used in a random response simulation of the oil pan system. Two cases were analyzed with different mass weighted composite modal damping applied to the system. The first case had a damping ratio 0.05 assigned to the steel with no effective damping ratio assigned to the oil. The second included a damping ratio of 0.02 for the oil in order to illustrate a technique that may be able to approximate the influence of fluid viscosity. The characteristics of the random response simulation model are:

- Unit pressure excitation applied to the entire pan outer surface
- DSLOAD implies random Load Case 1 (type = correlated)
- Excitation PSD was 0.01 for the frequency range 1 to 1001 Hz
- Case 1: Steel Composite Modal Damping = 0.05 (mass weighting)

  Oil Composite Modal Damping = 0 (no approximation of oil damping)
- Case 2: Steel Composite Modal Damping = 0.05 (mass weighting)

  Oil Composite Modal Damping = 0.02 (mass weighing approx.)
- Output response is the vertical velocity PSD at the bottom center of the pan.

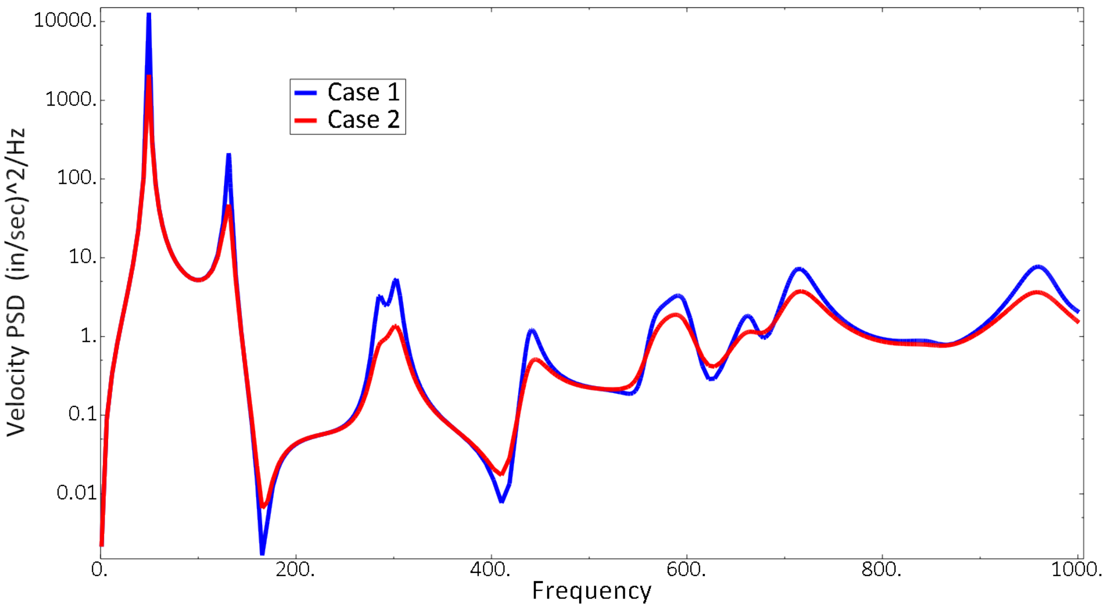


Figure 20: Oil Filled Pan Random Response Results

6. Avoidance of Modal Truncation Errors

The random response procedure uses modal superposition to create transfer functions that are then used to create the required PSD and RMS outputs. For excitation frequencies that are far below the first system resonance, the transfer functions of supported structures will have the characteristics of a spring. Also, for excitation frequencies centered between widely spaced resonances, the response behavior is predominantly stiffness controlled. Transfer functions generated by a direct solution procedure can capture this stiffness controlled behavior. However, when using modal superposition methods that truncate the number of modes utilized at an upper frequency limit, the accuracy of the transfer functions between the system resonances can be reduced. This in turn can reduce the accuracy of output PSD computations. Solution inaccuracies associated with modal truncation can be limited by including residual modes in the modal superposition, as discussed in the Abaqus Analysis Guide under the heading Natural Frequency Extraction.

To compute the residual mode associated with a loading condition, the frequency extraction step must be immediately preceded by a linear perturbation static step. Each load case defined in the static perturbation will correspond to a residual mode. Including the residual modes in a superposition will help limit truncation inaccuracies in Steady State and Random Response simulations. Residual modes should be included in the dynamic step superposition even if their frequencies are above the step's specified frequency range.

The issue of modal truncation errors will be demonstrated with the previously used cantilever beam example (see Figure 3). The excitation consists of three forces which are considered correlated with respect to random response concepts. The three forces are applied simultaneously, with their phasing unchanged in frequency. The three forces are all in the vertical direction, with 1 unit at the free end, 2 units at the mid-span and 3 units at ¼ span from the supporting wall. The demonstration will extract modes up to 2000 Hz, with Steady State and Random Response simulations performed over a frequency range of 0.01 Hz to 1000 Hz. Modal Damping with a 0.10 viscous damping ratio is used for all modes.

The modal truncation issue will first be demonstrated with Steady State Dynamic (SSD) procedures by determining the vertical reaction force (RF2) transfer function. For excitation frequencies well below the first resonant mode, the response is stiffness controlled. Therefore, the reaction force transfer function at the low end of the frequency range would be real-valued with a magnitude of 6 units.

Three SSD cases were solved:

- SSD-DIRECT – direct solution that does not have truncation errors
- SSD-MODAL – mode based solution (6 modes below 2000Hz)
- SSD-MODAL-RM – mode based solution (6 plus 1 residual mode)

The beam's mode shapes below 2000 Hz were shown in Figure 4. The beam frequencies below 2000 Hz are listed in Figure 21 along with a plot of the residual mode associated with the correlated load case.

Mode 1:	23.72 Hz
Mode 2:	148.43 Hz
Mode 3:	414.59 Hz
Mode 4:	809.61 Hz
Mode 5:	1332.4 Hz
Mode 6:	1979.7 Hz
Mode 7:	4035.0 Hz (Residual Mode)

Figure 21: Beam Model Residual Mode

The Steady State simulations provided 40 solutions between eigenmodes spaced with a bias factor of 2 on a LOG scale over the frequency range of 0.01 Hz to 1000 Hz. Figure 22 contains the reaction force transfer functions for the three simulation cases described above. It is clear that the addition of the residual mode in this case essentially eliminated the modal truncation error.

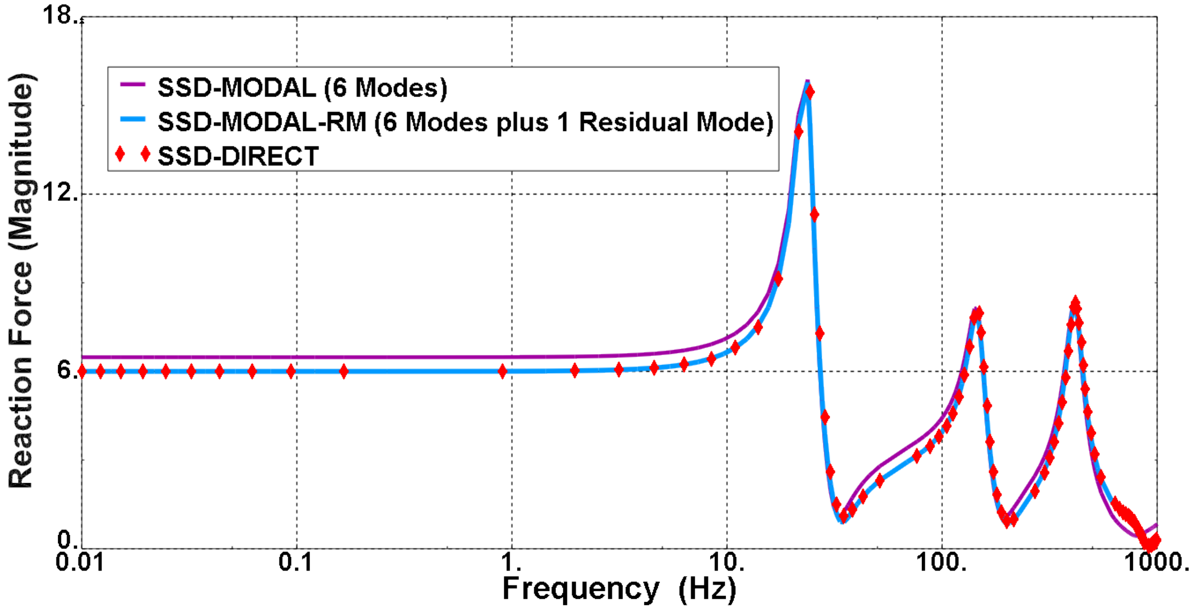


Figure 22: Steady State Dynamics Illustration of Modal Truncation Errors

The potential influence of the modal truncation issue on a random response simulation is demonstrated by determining the vertical reaction force output PSD and RMS response curves over the 0.01Hz to 1000 Hz frequency range. The PSD scalar function is defined with a value of 1 beginning at 0.001 Hz and remaining constant over the required frequency range. Two random cases were executed:

- RANDOM – Three CORRELATED Forces (without a residual mode)
- RANDOM-RM – Three CORRELATED Forces (with a residual mode)

As discussed earlier, the reaction force transfer function would have a magnitude of 6 units at the low end of the frequency range. Therefore, the associated reaction force PSD would have a value of 36, which is the transfer function squared times the PSD scalar function. The reaction force PSD and RMS curves are shown in Figure 23. Note: modal truncation errors can either increase or decrease the SSD and random response output, dependent on the modes being used in the superposition.

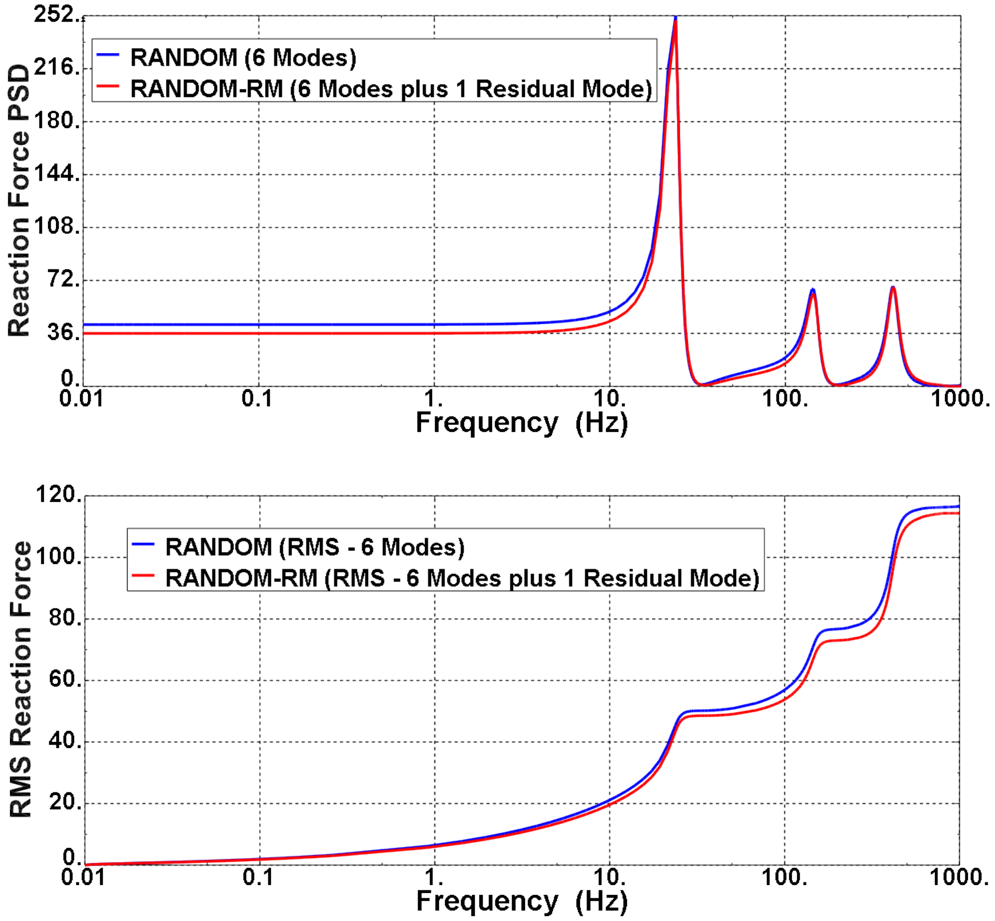


Figure 23: Reaction Force PSD and RMS Curves - Truncation Errors

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8. Document History

Document Revision	Date	Revised By	Changes/Notes
1.0	01/11/2018	David WOYAK, David PALMER	Original